Quantitative Methods for Safety Analysis in Aviation

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Acknowledgments

- Wayne Bryant, NASA Langley

Disclaimer: This tutorial solely represents the opinions of the author and does not necessarily reflect the opinion of the United States government.
Biography

- Assistant / associate professor of systems engineering at George Mason University (2000 – present)
- Previous experience with Qwest, US WEST (1997-2000)
- Education in mathematics, operations research
- Research interests
  - Methodology: Simulation, Queueing
  - Application areas: Aviation, Telecommunications
Outline

- Qualitative and quantitative methods
- A primer on probability
- Measurements of safety
  - Safety metrics
  - Comparing measurements of safety
- Some classical safety methods
  - Fault trees
  - Event trees
- Models of aircraft separation
- Collision modeling
- Data requirements
Safety Assessment Techniques

• 500+ techniques identified in Eurocontrol study
• ~100 techniques identified in FAA System Safety Handbook

Safety Analysis Methods

Qualitative Methods
- Fishbone
- Bow-Tie
- Expert Judgment
- ORR
- FMECA
- HAZOP

Quantitative Methods
- Fault Tree
- Simulation
- Statistics
- Event Tree

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Focus / Scope of Tutorial

• Quantitative methods
  – The output is a number (on a continuous scale)
  – Confidence in the output can be quantified

• Congestion-related safety
  – Separation violations
  – Collisions
  – Wake encounters

• “Pre-requisites”
  – Working knowledge of concepts from probability and statistics
Qualitative / Quantitative Methods

System Description

Hazard Analysis

Initial Hazard Ranking

Model A

Model B

Not Modeled

Quantitative Analysis

Safety Metrics

Qualitative Analysis

Quantitative Modeling
Why Use Quantitative Methods?

- Greater precision in safety analysis
- Determine if system meets quantitative standard
- Determine design requirements
  - What degree of accuracy is provided by wind sensors?
  - How many wind sensors are needed?
- Determine safety envelop of system
  - How close can parallel runways be to safely implement the proposed procedures?

Problems with Quantitative Methods

- Difficult to describe the system mathematically
- Difficult to describe relevant safety hazards – and their consequences – in a mathematical model
- Not enough data to estimate model parameters
- User trust in numerical output is too high
  – Confidence level in output is missing

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Probability Density Functions

Histograms:
- 100 points
- 500 points
- 5,000 points

Infinite points

All Graphs Notional

Probability Density Function
Probability Density Functions

Probability that time separation is between 100 and 140 seconds is the area under the curve

\[
\text{Pr}(100 \leq \text{time sep} \leq 140) = \int_{100}^{140} f(t) \, dt
\]
Some Common PDF’s

Form of PDF

Normal

\[ f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \]

Gamma

\[ f(x) = \frac{1}{\Gamma(\alpha) \beta^x} x^{\alpha-1} e^{-x/\beta} \]

Exponential

\[ f(x) = \lambda e^{-\lambda x} \]

Mean = 106
Std. Dev. = 26.9 (normal and gamma)
Summary

- Data are often collected in “bins” and displayed using a histogram.
- Histograms can be approximated by probability density functions (PDF).
  - “Bin” sizes are infinitely small.
- PDF’s can be more mathematically convenient to do analysis.
- Common misuses in this process:
  - Amount of data does not justify PDF approximation.
  - Extrapolation of events beyond what is observed.
  - A well-known PDF (e.g., normal) is assumed when data do not justify it.

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Levels of Risk

Sources

# Deaths / Hours of Exposure
UK, 1992

- Rock Climbing
- Travel by Air
- Travel by Car
- Accident at Home (able-bodied)
- Building Collapse
## NTSB 2005 Preliminary Statistics
### U.S. Civil Aviation Accidents, Fatalities and Rates

<table>
<thead>
<tr>
<th>Sector</th>
<th>Accidents</th>
<th>Fatalities</th>
<th>Flight Hours</th>
<th>Incidents / 100,000 Flight</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>Fatal</td>
<td>Total</td>
<td>On board</td>
</tr>
<tr>
<td><strong>Part 121 Carriers</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scheduled</td>
<td>32</td>
<td>3</td>
<td>22</td>
<td>20</td>
</tr>
<tr>
<td>Non-scheduled</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Part 135 Carriers</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commuter</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>On-Demand Taxi</td>
<td>66</td>
<td>11</td>
<td>18</td>
<td>16</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1669</td>
<td>321</td>
<td>562</td>
<td>557</td>
</tr>
</tbody>
</table>

http://www.ntsb.gov/aviation/Stats.htm
Different Metrics

U.S. Part 121, 1986 – 2005

- Fatal accidents per flight hour: $2.6 \times 10^{-7}$
- Fatal accidents per mile flown: $6.3 \times 10^{-10}$
- Fatal accidents per departure: $4.0 \times 10^{-7}$
- Passenger fatalities per departure: $1.1 \times 10^{-5}$
- Passenger fatalities per enplanement: $1.9 \times 10^{-7}$

1. Fatal accident rates do not include illegal acts (e.g., terrorism). However, passenger fatality rates do include illegal acts, but they exclude crew deaths.
2. Accident statistics are averages of yearly averages, passenger fatalities are averages across 1986-2005.

http://www.ntsb.gov/aviation/Stats.htm
Different Types of Operations

Fatal Accidents per Flight Hour, 1986 – 2005

- Part 121: $0.026 \times 10^{-5}$
- Part 121, Scheduled: $0.022 \times 10^{-5}$
- Part 121, Non-scheduled: $0.76 \times 10^{-5}$
- Part 135, Commuter: $0.27 \times 10^{-5}$
- Part 135, On-demand: $0.80 \times 10^{-5}$
- General Aviation: $1.5 \times 10^{-5}$

http://www.ntsb.gov/aviation/Stats.htm
Some Problems with Metrics

Fatal accidents per flight hour

A crash in which 1 out of 100 people killed

equally serious as

A crash in which 100 out of 100 people killed

/  
Total # of flights with at least one fatality

/  
Total # of flight hours

/  
Crashes on long flights less serious

than crashes on short flights

Some Problems with Metrics

Passenger Fatalities per Enplanement

\[
\frac{\text{Total # of passenger fatalities}}{\text{Total # of passenger enplanements}}
\]

Problem: A 150-seat airplane flies into a mountain. If the airplane happens to have 150 on board, this is 3 times as bad as if 50 people are on board (that is, the metric does not account for probability of death on the airplane)

An Alternative Metric

• What is the probability that I am killed on a flight?
  – I randomly choose a flight
  – I randomly pick a seat on the flight
  – What is probability that I am killed?

\[
\frac{\sum_{i=1}^{N} k_i / n_i}{N}
\]

Number of Passengers killed on flight \(i\)  \hspace{3cm} Number of Passengers on flight \(i\)

\(k_i\) \hspace{5cm} \(n_i\)

\(\sum_{i=1}^{N}\) \hspace{5cm} \(N\)

Total Number Of flights

Sample of Metric

• Passenger death risk per US domestic jet flight, 1987-96, was about 1 in 7 million
• Taking one flight per day, it would take (on average) 19,000 years to be killed in a flight.

A New(?) Metric

• What is the probability that I am killed on a flight?
  – I randomly choose a flight, proportional to number of passengers on flight
  – I randomly pick a seat on the flight
  – What is probability that I am killed?

\[
\sum_{i=1}^{N} \left( \frac{n_i}{\sum n_i} \right) \left( \frac{k_i}{n_i} \right) = \frac{\sum_{i=1}^{N} k_i}{\sum_{i=1}^{N} n_i} = \text{passenger fatalities per enplanement}
\]

- \( n_i \) = number of passengers on flight \( i \)
- \( k_i \) = number of passengers killed on flight \( i \)
Summary

- No metric is perfect
- “Fatal accidents / flight hour” is not the best metric
- All safety statistics suffer from rare event problem
  - More on this later
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Sample Comparison

• “General aviation aircraft were involved in 52 more accidents [in 2005] than in 2004, 321 of which were fatal – an increase from 314. The GA accident rate increased from 2004’s 6.49 per 100,000 flight hours [to 6.83 in 2005]. … While the accident increase is “disappointing,” … total fatalities for 2005 decreased to 600 from 636.”

Sample Comparison

Fatal Accidents per Flight Hour:
General Aviation and Part 121 (all)

Note: Scale is different for GA & Part 121

95% Confidence Interval for Part 121 (all) 2005

95% Confidence Interval for GA 2005

Did things really get worse Last year?

http://www.ntsb.gov/aviation/Stats.htm
Another View

95% Confidence Intervals
Fatal Accidents per Flight Hour

- Part 121, 2004
- Part 121, 2005
- General Aviation, 2004
- General Aviation, 2005
Example

- A graduate student observes one simultaneous runway occupancy (SRO) out of 245 landings.
- An alternate source states that the probability is one out of 5000 landings.

Are these findings consistent?
Poisson Random Variables

- The sum of a large number of independent, rare events approximately follows a Poisson distribution.
- Specifically, let

\[ X_i = \begin{cases} 
1 & \text{with probability } p_i \\
0 & \text{with probability } 1 - p_i
\end{cases} \]  

(individual events)

\[ X \equiv \sum_{i=1}^{N} X_i \]  

(total count of events)

\[ p \equiv \sum_{i=1}^{N} p_i \]  

(expected total number of observed events)

\[ P(X = i) \approx e^{-p} \frac{p^i}{i!} \]  

(Poisson distribution)
Poisson Distribution

- Ladislaus Bortkiewicz, 1898
- Data
  - 10 Prussian army corps units observed over 20 years (200 data points)
  - A count of men killed by a horse kick each year by unit
  - Total observed deaths: 122
- Number of deaths (per unit per year) is a Poisson RV with mean \( \frac{122}{200} = 0.61 \).

<table>
<thead>
<tr>
<th>Number</th>
<th>Theoretical</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>108.67</td>
<td>109</td>
</tr>
<tr>
<td>1</td>
<td>66.29</td>
<td>65</td>
</tr>
<tr>
<td>2</td>
<td>20.22</td>
<td>22</td>
</tr>
<tr>
<td>3</td>
<td>4.11</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>0.63</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0.08</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0.01</td>
<td>0</td>
</tr>
</tbody>
</table>
Poisson Confidence Interval

• Definitions
  – Let $X$ be one observation of a Poisson random variable (a count of events)
  – Let $\theta$ be the true mean of the random variable
  – Let $H_k(x)$ be the CDF of a $\chi^2$ distribution with $k$ degrees of freedom.
  – Let $(1 – \alpha)$ be the desired confidence (e.g., $\alpha = 0.05$ is a 95% confidence interval)

• Using $X$ to estimate $\theta$, a confidence interval for $\theta$ is

$$\left[ H^{-1}_{2X}(\alpha / 2) / 2, \quad H^{-1}_{2X+2}(1 – \alpha / 2) / 2 \right]$$

• Bounds in Excel:
  ‘= CHIINV( 1 – <\alpha>/2, 2*<X> ) / 2’ (Lower)
  ‘= CHIINV( <\alpha>/2, 2*<X> + 2 ) / 2’ (Upper)

CDF = Cumulative Distribution Function (In Excel, CHIINV function inverts the complementary CDF)
• Let $X$ be the number of SRO’s observed out of 245 landings ($X$ is an integer between 0 and 245).
• Let $\theta$ be the true probability for an SRO.
  – The point estimate for $\theta$ is $X / 245$.
  – The true expected number of observed SRO’s is $(245 \theta)$
• If $X = 1$, then 95% confidence interval for $(245 \theta)$ is $[0.03, 5.57]$
• CI for $\theta$ is $[1 \times 10^{-4}, 0.02]$
Another Example

Safety Comparison
Domestic Flights, 1987 – 1996

<table>
<thead>
<tr>
<th>Airline</th>
<th>Domestic Full-Crash Equivalents</th>
<th>Number of Flights (millions)</th>
<th>Probability of Being Killed on a Random Flight</th>
</tr>
</thead>
<tbody>
<tr>
<td>American</td>
<td>0.00</td>
<td>7.2</td>
<td>--</td>
</tr>
<tr>
<td>Continental</td>
<td>0.32</td>
<td>4.5</td>
<td>7.1E-08</td>
</tr>
<tr>
<td>Delta</td>
<td>0.16</td>
<td>8.5</td>
<td>1.9E-08</td>
</tr>
<tr>
<td>Northwest</td>
<td>1.21</td>
<td>4.9</td>
<td>2.5E-07</td>
</tr>
<tr>
<td>TWA</td>
<td>0.00</td>
<td>2.7</td>
<td>--</td>
</tr>
<tr>
<td>United</td>
<td>1.40</td>
<td>6.5</td>
<td>2.2E-07</td>
</tr>
<tr>
<td>US Air</td>
<td>3.53</td>
<td>8.6</td>
<td>4.1E-07</td>
</tr>
</tbody>
</table>

Is (was) US Air unsafe?

Another Example (cont.)

- Comparison: Probability of being killed on a random flight
  - US Air: 1 in 2.4 million
  - Other carriers: 1 in 11.6 million

- Assuming all flights are equally safe, there is an 11% chance that one airline would fare as badly as US AIR over this time period
  - Not statistically significant in usual sense (but close)

- Was US Air less safe than other carriers (combined)?
  - From 1987 – 1996? YES
  - Looking forward? NOT in a statistically significant way

Summary

- Safety statistics usually specify an *observed* frequency of bad events
  - Be wary of interpreting the observed frequency as the “true” probability of a bad event
- For rare events, confidence intervals may be several orders of magnitude in size
- Without confidence intervals, be wary of comparisons between safety statistics
- Safety metrics are often “order of magnitude”
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Fault Trees

• Top-down analysis

• System failure is postulated (top event)
  • System failure described as logical function of sub-system failures
  • Sub-system failures described as logical function of sub-sub-system failures
  • Breakdown continues until
    – Data are available for bottom level events, or
    – Scope of problem exhausted

• Goal: Define all possible failure combinations that lead to system failure
Some Standard Elements

Logic Gates
AND gate

OR gate

Input Events
Basic Event

Undeveloped Event

Other
Transfer

Comment Rectangle
Basic Operations

**AND gate**

- No toothbrush
- Forget toothbrush
- Wife does not remind me

\[ p = \text{Prob}(E) \]
\[ p = p_1 \cdot p_2 \]

\[ p_1 = \text{Prob}(E_1) \]
\[ p_2 = \text{Prob}(E_2) \]

**OR gate**

- Miss my flight
- Arrive Late
- Forget Passport

\[ 1 - p = (1 - p_1) \cdot (1 - p_2) \]
\[ = 1 - p_1 - p_2 + p_1 p_2 \]

\[ p = p_1 + p_2 - p_1 p_2 \]
Sample Calculation

Problem Setup:
- System works if 3 sub-systems are \textit{simultaneously} working
- Each sub-system is dual redundant
- Individual sub-system failure probabilities are 0.1

\[
1 - (.99)(.99)(.99) = .029701
\]

This calculation works if:
- Component failures are independent
- Each component appears only once in fault tree
Alternate System Design

Problem Setup:

- **System** is dual redundant
  - As before, each “system” works if 3 sub-systems simultaneously working

\[
.0734 = (.271)(.271) > 0.0297
\]

1 – (.9)(.9)(.9) = 0.271

General Principle: Better to use redundancy at component level than system level
Example: 2 out of 3 System

- System is working if (at least) 2 out of 3 components are working.
- Equivalently, system is failed if (at least) 2 out of 3 components are failed.
Definitions

• Component indicator

\[ X_i(t) = \begin{cases} 1 & \text{if component } i \text{ is failed at time } t \\ 0 & \text{if component } i \text{ is working at time } t \end{cases} \]

• Component failure probability

\[ p_i(t) = P(X_i(t) = 1) \]

• Structure function (system indicator)

\[ \phi(X(t)) = \begin{cases} 1 & \text{if system is failed at time } t \\ 0 & \text{if system is working at time } t \end{cases} \]
Structure Function

\[ \phi(X) = 1 - (1 - X_1 X_2)(1 - X_1 X_3)(1 - X_2 X_3) \]

\[ = X_1 X_2 + X_1 X_2 + X_1 X_2 - X_1^2 X_2 X_3 - X_1 X_2^2 X_3 - X_1 X_2 X_3^2 + X_1^2 X_2 X_3^2 \]

\[ = X_1 X_2 + X_1 X_2 + X_1 X_2 - X_1 X_2 X_3 - X_1 X_2 X_3 - X_1 X_2 X_3 + X_1 X_2 X_3 \]

\[ = X_1 X_2 + X_1 X_2 + X_1 X_2 - 2 X_1 X_2 X_3 \]

\[ p_S = p_1 p_2 + p_1 p_3 + p_2 p_3 - 2 p_1 p_2 p_3 \]
Suppose probability of individual component failure is 0.01

\[ p_s = p_1 p_2 + p_1 p_3 + p_2 p_3 - 2 p_1 p_2 p_3 \]
\[ = .01 + .01 + .01 - .002 \]
\[ = .028 \]

(Answer is not 1 – (.99)(.99)(.99) = .029701, though it’s close)
ADS-B Example

System Description

- Scope: Final approach
- Flight crews given speed guidance to maintain spacing behind a lead aircraft
- Information transmitted from lead aircraft via ADS-B.
- Goal: Reduce inter-arrival variance at runway threshold

Figure 4. High level fault tree for wake vortex encounter analysis.

Figure 5. Operational/system errors lead to path that violates WV separation minima.

Fault Tree Example

Figure 6. Fault tree for misleading guidance.

Probabilities and Failure Rates

• Two ways to describe the probability of an event
  • Probability $Q$
  • Failure rate $r$

• Common assumptions
  • Failure rate is constant over the time interval of interest
  • Failed components remain failed over time interval of interest.

• Relationship – Probability of a failure before $t$:
  \[ Q = 1 - e^{-rt} \]

• If $rt$ is small, then $Q = 1 - e^{-rt} \approx rt$
  (that is, probability of a failure is roughly the failure rate times the time horizon)
Advantages

- Fault trees are useful for reliability analysis, particularly for hardware
- A (small) fault tree is easy to read and understand
- Fault trees quickly expose critical paths
- Well accepted

Eurocontrol. 2004. Review of techniques to support the EATMP safety assessment methodology. EEC Note No. 01/04 – Volume I.
Drawbacks

• Dynamic aspects are difficult capture. Most naturally, a fault tree is a *snapshot* of the system in time
  – Sequence of events matters
  – Duration of events matters
  – Time when event occurs matters

• Potential for abuse: Final quantitative result hides lack of data / confidence in low level probabilities

EEC Note No. 01/04 – Volume I.
Event Trees

- Initial failure / hazard is postulated
- Future events / failures / mitigations and associated probabilities are described

Possible causes
- Pilot error
- System error
- Adverse weather

Diagram:
- Heading error
  - Heading error undetected
  - Heading error detected
  - Trailer Exposed
    - Loss of Control
      - Accident
      - No Accident
Summary

• Fault trees and event trees are widely used and accepted methods.
• Generally easy to understand
• Useful in high-level analysis
• Potential difficulties in detailed air-traffic modeling, where sequence, duration, and timing of events matters.
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Direct Observation

- Observed SRO frequency: 3 out of 1862 $\approx 0.0016$ (in peak periods)
- 95% Confidence interval: [0.0003, 0.005]

**SRO**: Simultaneous Runway Occupancy

Fitted Distributions

Inter-Arrival Time (sec)

Runway Occupancy Time (sec)

Theoretical Method

\[ P(LTI < ROT) = \int_0^\infty P(LTI < ROT \mid ROT = x) f_{ROT}(x) \, dx \]

\[ = \int_0^\infty \int_0^\infty f_{LTI}(y) f_{ROT}(x) \, dy \, dx \approx 0.004 \]

Confidence Intervals

\[
\text{E}(\text{p}(\text{LTI}<\text{ROT})) = 0.0035 \\
97.5\% \text{ CI} \approx [0.00014, 0.0359]
\]

(3 nm – separated aircraft)

Theoretical Method

• Advantage: Can test “what if?” by varying mean and standard deviation

• Disadvantage: Extra assumptions:
  – Randomness modeled by given distributions
  – ROT and IAT are independent
SRO Sensitivity to Separation

Std.Dev. (Sep.)
(sec.)

Safer

Mean Sep. (sec.)

SRO: Simultaneous Runway Occupancy

Further Extensions

Examples of Derived Statistics
A. Time / dist between two aircraft
B. Lateral deviation of aircraft from centerline
C. Vertical deviation of aircraft from glide-slope
D. Velocity of aircraft
E. Runway occupancy time

Measured At:
Threshold, 1 nm out, 2 nm out, …
Cross-Section Scatter Plots

6 nm (525 pts)

5 nm (1156 pts)

4 nm (1179 pts)

3 nm (1188 pts)

2 nm (1188 pts)

1 nm (1190 pts)

0 nm (1190 pts)
Vertical Positions

Fitted Distributions
Dynamically Colored Petri Nets

Airplane $i$

Airplane $j$

Runway

Threshold

2 nm  1 nm

$T_1$  $T_2$  $T_3$

Airplanes
Phases of Flight
Transitions

Everdij, M., H. Blom, M. Klompstra. 1997. Dynamically coloured Patri nets for air traffic management safety purposes. 8th IFAC Symposium on Transportation Systems, Chania Greece, NLR report TP 97493 L.
Airplane Trajectory

Airplane trajectory in each phase is a solution to a set of *stochastic* differential equations

\[
\begin{align*}
\dot{p}_y &= v_y \\
\dot{v}_y &= -av_y - b(p_y - y_0) + \text{“noise”}
\end{align*}
\]

Lateral Position

Pilot tries to stay on runway center line

Equipment Accuracy

Wind

Pilot Workload

Pilot tries to stay on runway center line
### Landing Sequence

#### Markov Chain:

![Markov Chain Diagram](image)

#### Transition Probability Table:

<table>
<thead>
<tr>
<th>Leader</th>
<th>Heavy</th>
<th>Large</th>
<th>B757</th>
<th>Small</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heavy</td>
<td>0.0313</td>
<td>0.5</td>
<td>0.25</td>
<td>0.2187</td>
</tr>
<tr>
<td>Large</td>
<td>0.0702</td>
<td>0.6198</td>
<td>0.124</td>
<td>0.186</td>
</tr>
<tr>
<td>B757</td>
<td>0.1053</td>
<td>0.5965</td>
<td>0.1404</td>
<td>0.1578</td>
</tr>
<tr>
<td>Small</td>
<td>0.1096</td>
<td>0.5616</td>
<td>0.1507</td>
<td>0.1781</td>
</tr>
</tbody>
</table>

Input Distributions

Arrival Sequence

Small, Large, Heavy, B757

Final Approach

Inter-arrival time

Runway Occupancy Time

On Runway

T₁

Off Runway

T₂

Decompose ROT

Runway Occupancy Time (seconds)

PDF

0.1
0.05
0.05
0

Small
Large
B757
Heavy

Decompose Landing Time Interval

Histogram Landing Time Interval; classes A and B; Rush times (>=7arr/qtr) - IMC

3 nm separation

4 nm separation

Added Features

Can test

• Changes to fleet mix
• Changes to separation matrix
Expected Distribution of SRO’s

- Large-Large
- Large-Heavy
- Large-B757
- B757-Large

Leader - Trailer

Extensions: Hazard Modeling

Critical Parameters: Probability of Missed Approach
Example Model

Basic Scenario Modeled
Simultaneous missed approach. Aircraft on runway 22 fails to turn.

Blom, H., M. Klompstra, B. Bakker. 2001. Accident Risk Assessment of Simultaneous Converging Instrument Approaches. 4th USA / Europe ATM R&D Seminar, Sante Fe, NM.
Example Model

Parallel Activities: Climb & Reconfiguration

Go Around?

Event Sequence Logic: Timing of Turn

Blom, H., M. Klompstra, B. Bakker. 2001. Accident Risk Assessment of Simultaneous Converging Instrument Approaches. 4th USA / Europe ATM R&D Seminar, Sante Fe, NM.
Reliability, Human Factors

Wake Monitor

- Working
- Not Working

Pilot Cognitive State

- Busy
- Relaxed
- Frantic

Here, token • represents some system state
Outline

• Qualitative and quantitative methods
• A primer on probability
• Measurements of safety
  – Safety metrics
  – Comparing measurements of safety
• Some classical safety methods
  – Fault trees
  – Event trees
• Models of aircraft separation
• Collision modeling
• Data requirements
Basic Problem
Predicting Collisions

First assume a cubical airplane…

Airplane wingspan and length are approximately similar
Definition of Collision Area

$$\lambda_x \quad \lambda_y \quad \lambda_z$$

Plane A

Plane B

$$\vec{x}_A - \vec{x}_B$$

Collision Area

$$2\lambda_x \quad 2\lambda_y \quad 2\lambda_z$$
Definition of Collision Area

Relative position of two airplanes

$$\bar{x}_r(t) = \bar{x}_A(t) - \bar{x}_B(t)$$

Collision occurs whenever $$\bar{x}_r(t)$$ enters box
Reich Collision Model

- Collision rate definition
  \[ \phi(t) = \lim_{\Delta t \to 0} \frac{P(\bar{x}_t \in D, \bar{x}_{t-\Delta t} \notin D)}{\Delta t} \]

- Reich model
  \[ \phi(t) = N_x P_y P_z + N_y P_x P_z + N_z P_x P_y \]

\[ P_y = P(|y_t| \leq \lambda_y) \]
\[ P_z = P(|z_t| \leq \lambda_z) \]
\[ N_x = \lim_{\Delta t \to 0} \frac{P(|x_t| \leq \lambda_x, |x_{t-\Delta t}| > \lambda_x)}{\Delta t} \]

Assumptions

- Collision area is a box
- $x$, $y$, $z$ components all independent
- No system feedback

- Often assumed:
  - Velocity and position are independent
  - Density function constant over collision region

Illustration of Model in 1-D

$N_x = \text{rate of entering collision area}: \quad N_x = \frac{v_x}{B}$

$P_x = \text{Fraction of time in collision area}: \quad P_x = \frac{2\lambda_x}{B}$

$N_x = \frac{v_x P_x}{2\lambda_x}$
Example Application

\[ \phi = N_x P_y P_z + N_y P_x P_z + N_z P_x P_y \]

\[ N_y = \frac{|v_y| P_y}{2\lambda_y} \quad P_x = \frac{2\lambda_x}{|v_x|} N_x \]

\[ N_z = \frac{|v_z| P_z}{2\lambda_z} \]

\[ \phi = N_x P_y P_z \left( 1 + \frac{|v_y| \cdot \lambda_x}{|v_x| \cdot \lambda_y} + \frac{|v_z| \cdot \lambda_x}{|v_x| \cdot \lambda_z} \right) \]

Key parameter to estimate
Example Application

\[ \phi = N_x P_y P_z \left( 1 + \frac{|v_y| \lambda_x}{|v_x| \lambda_y} + \frac{|v_z| \lambda_x}{|v_x| \lambda_z} \right) \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral path-keeping standard deviation</td>
<td>( \sigma_y ) 550 m</td>
</tr>
<tr>
<td>Lateral overlap probability</td>
<td>( P_y ) 0.058</td>
</tr>
<tr>
<td>Vertical overlap probability</td>
<td>( P_z ) 1.7 x 10^{-8}</td>
</tr>
<tr>
<td>Opposite-direction passing frequency</td>
<td>( N_x \text{ (opp)} ) --</td>
</tr>
<tr>
<td>Same-direction passing frequency</td>
<td>( N_x \text{ (same)} ) --</td>
</tr>
<tr>
<td>Average aircraft length</td>
<td>( \lambda_x ) 45 m</td>
</tr>
<tr>
<td>Average aircraft width</td>
<td>( \lambda_y ) 45 m</td>
</tr>
<tr>
<td>Average aircraft height</td>
<td>( \lambda_z ) 15 m</td>
</tr>
<tr>
<td>Average relative same-direction aircraft speed</td>
<td>(</td>
</tr>
<tr>
<td>Average relative opposite direction aircraft speed</td>
<td>(</td>
</tr>
<tr>
<td>Average relative cross-track aircraft speed</td>
<td>(</td>
</tr>
<tr>
<td>Average relative vertical aircraft speed during loss of separation</td>
<td>(</td>
</tr>
</tbody>
</table>

ICAO. 2002. Manual on implementation of a 300m (1000 ft) vertical separation minimum between FL 290 and FL 410 inclusive. 2nd Ed. Doc 9574.
Sample Calculation

• Opposite direction traffic

\[ \phi(t) = N_x P_y P_z (1.000 + 0.004 + 0.033) \]
\[ x\text{-face} \quad y\text{-face} \quad z\text{-face} \]

• Same direction traffic

\[ \phi(t) = N_x P_y P_z (1.000 + 0.189 + 1.541) \]
\[ x\text{-face} \quad y\text{-face} \quad z\text{-face} \]
Generalizations

- Generalized Reich Model (Blom 1993)

\[ \phi(t) = \sum_{i=1}^{3} \int_{\bar{p} \in S_i^{-}} \int_{\bar{v} \in \mathbb{R}_i^{3}} \int_{\bar{p} \in S_i^{+}} \int_{\bar{v} \in \mathbb{R}_i^{3}} v_i f_t(\bar{p}, \bar{v}) \, d\bar{v} \, d\bar{p} + \int_{\bar{p} \in S_i^{-}} \int_{\bar{v} \in \mathbb{R}_i^{3}} \int_{\bar{p} \in S_i^{+}} \int_{\bar{v} \in \mathbb{R}_i^{3}} v_i f_t(\bar{p}, \bar{v}) \, d\bar{v} \, d\bar{p} \]

- Generalized Reich Model (Shortle 2005)

\[ \phi(t) = \int_{\bar{x} \in S} \int_{\bar{v} \in \mathbb{R}^{3}} (\bar{v}^T \cdot \bar{n}(\bar{x}))^+ f_t(\bar{x}, \bar{v}) \, d\bar{v} \, dS \]

Assumptions
- Collision area is a box
- \( x, y, z \) components all independent
- No system feedback
Application in Modeling

Relative position of two airplanes
\[ \vec{x}_r(t) = \vec{x}_1(t) - \vec{x}_2(t) \]

\[ \phi(t) = \sum_{i=1}^{3} \int_{\vec{p} \in S_i^-} \int_{\vec{v} \in \mathbb{R}_i^3} v_i f_i(\vec{p}, \vec{v}) d\vec{v} d\vec{p} + \int_{\vec{p} \in S_i^+} \int_{\vec{v} \in \mathbb{R}_i^3} v_i f_i(\vec{p}, \vec{v}) d\vec{v} d\vec{p} \]

Output from simulation:
Distribution of relative position of airplane pair

Expected # Collisions
\[ \int_{a}^{b} \phi(t) dt \]

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Key Questions

• How much data are needed to do quantitative safety analysis?
• Can you really estimate a $10^{-9}$ event?
Can You Estimate a $10^{-9}$ Event?

- Yes: Flip a coin 30 times.
- The probability of 30 heads in a row is 1 in $2^{30}$ or $0.93 \times 10^{-9}$. 
What if you Don’t Know $p$?

Problem
What is the probability of flipping $M$ heads in a row when you don’t know the true probability $p$ of heads?

Basic algorithm
• Flip a coin $N$ times.
• Let $\hat{p} = \frac{\text{# heads}}{N}$
• Estimate for rare event is $\hat{p}^M$
Coin Flipping Problem

Sample problem: $p = 0.4365$, $M = 25$, $p^M = 10^{-9}$

For this problem, about $10^4 – 10^5$ trials are needed to accurately estimate the rare event probability $10^{-9}$
Key Insights

- Multiple “semi-rare” events combine to yield the rare event.
- Data requirements for multiple “semi-rare” events are less than for one rare event.

Qualifying Remarks / Assumptions

- The “semi-rare” events (in example) are independent
  - Correlation reduces, but does not eliminate, benefits
- The “semi-rare” events have equal probabilities
  - Benefits reduced for asymmetric probabilities
- The model linking “semi-rare” events to the rare event is believable
Overlap Probability Calculation

- Count number of observations in tail. 5
- Estimate upper bound on tail probability via Poisson confidence interval 11.7 / 10,000
- Choose the form of the tail shape Uniform
- Based on upper bound for tail probability, compute overlap probability

Sample Calculation

Nominal Vertical Separation: 1000 ft
Average Airplane Height: 50 ft

Data Requirements

Basic Question: How much data are needed for a good estimate of the overlap probability?

1. Let $p^*$ be the true overlap probability
2. Let $c$ be a confidence level (e.g., $c = 0.99$)
3. Let $P$ be the conservative estimate for the overlap probability, based on making $N$ observations.
   \[ P(p^* < P) = (1 + c) / 2 \]
4. Determine $k$ such that:
   \[ P(p^* < P < k \cdot p^*) = c \]
Data Requirements

Hypothesized True Overlap Probability = 10^{-9}

Number $N$ of Data Points

Exponential Tails

Uniform Tails

CI = 80%
CI = 90%
CI = 95%
CI = 99%
Extension of Reich Model to Wakes

Physics included:
- Probabilistic estimate of wake demise
- Wake sink
Extension of Reich Model to Wakes

Physics included:
• *Conservative* estimate of wake demise
Data Requirements

Hypothesized True Overlap Probability = 10^{-9}

Data requirements less if trying to predict less-rare event (10^{-6})

Notional results that depend on independence of variables, and other specific assumptions
Bias & Uncertainty Assessment

Evaluates bias and uncertainty based on model assumptions, parameter values, and omissions.

- Blom, H., M. Klompstra, B. Bakker. 2001. Accident Risk Assessment of Simultaneous Converging Instrument Approaches. 4th USA / Europe ATM R&D Seminar, Sante Fe, NM.
Summary

• The tails of the distributions matter
  – Knowing the body of the distribution does not imply knowing the tail
  – High level of incidents does not necessarily mean high level of accidents
  – Extrapolation from incidents to accidents should be justified statistically

• Data requirements for rare events with several independent contributory events may be achievable
Overall Summary

• Quantitative methods are an important subset of methods for safety assessment
• Several approaches discussed
  – Direct measurement
  – Fault trees / events trees
  – Simulation modeling
  – Collision modeling
• Confidence levels are important for rare events