

Air Transportation Systems Engineering

Delay Analysis Workbook

Air Transportation Delay Analysis Workbook

Actions:

1. Read Chapter 23 Flows and Queues at Airports
2. Answer the following questions.

Introduction

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A generic queuing system consists of three components:

1. _____
2. _____
3. _____

Each user of the queuing system is generated by the _____, passes through the _____ where it remains for some time period (zero to t), and then is processed by one of the parallel _____.

A queuing network is a set of _____. In a queuing network, the user sources for some of the queuing systems maybe _____.

User Generation Process

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Two properties of the User Generation Process:

1. The _____ is the rate at which the users arrive over time. The Greek letter _____ is used to denote this parameter.

Example demand rate: The rate at which flights arrive at La Guardia airport during the peak period of operations is 8 flights per 15 minutes.

2. The _____ is the time interval between successive demands. This time interval is referred to as the _____ or shortened to _____

Example probability distributions: (1) the flights arrive at the runway evenly spaced (i.e. one flight every 90 seconds), (2) the flight arrive on average once every 90 seconds, but distributed randomly (and independently) in time.

The probability distribution for the 1st example is known as _____ (or _____) demand.

The probability distribution for the 2nd example is known as _____

The process of random, independent distribution is known as the _____ Process. This process exhibits a negative exponential probability density function.

Draw a negative exponential probability density function. The x-axis is Demand Rate Inter-arrival Time, the y-axis is Probability Density



Short demand inter-arrival time occur with high/low probability. (Circle one).

Long-demand inter-arrival times occur with high/low probability (Circle one)

The expected value (average) length of the demand inter-arrival is equal to $1/\lambda$ which is equal to the _____ demand rate.

Describe in qualitative terms what happens with Poisson probability distributions.

Which type of system is more likely to experience delays – arrival rate with deterministic inter-arrival time distribution, or an arrival rate with Poisson inter-arrival time distribution _____?

Explain why _____

Aside: Why do the Washington DC Metro stations have 2 escalators from platform-to-street and only 1 escalator from street-to-platform? Explain.

Service Processes

Page 823

The service rate is the _____ per unit time. This parameter is represented by the Greek letter _____.

Example. The service rate for a runway is 12 flights per 15 minutes in Visual Meteorological Conditions (VMC) and 10 flights in Instrument Meteorological Conditions.

The probability distribution that describes the duration of service times is known as _____

Example: The probability distribution for service times of a runway is known as Runway Occupancy Time (ROT). ROT varies by aircraft type. A typical ROT has $\mu = 45$ seconds, and $\sigma = 8$ secs.

Queuing Process

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Most queues for aircraft at airports operate on a First-Come/First-Serve (FCFS) discipline.

Explain FCFS _____

Constrained Position Switch (CPS) is an alternate queuing discipline. Explain what CPS is and how it works.

A crucial parameter in describing and designing airport queuing systems is queue capacity. This is the _____

Examples of queue capacity for flights:

1. Departure Runway queues are limited by _____

2. Arrival Runway queues are limited by _____

Measures of Performance and Level of Service

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Utilization Ratio, also known as _____ ratio.

Represented by Greek letter _____

Utilization Ratio is defined as _____ / _____

When $\rho > 1$ system is considered to be _____ and delays are _____

When $\rho = 1$, delays are _____

When $\rho < .78$, delays are _____

Waiting Time, represented by _____, is defined as _____

Number of Users in the queue, represented by _____, is defined as _____

Waiting Time and Number of Users in queue are _____ variables

because demand interval times and service times are _____

As a consequence the Expected Values of these parameters are used.

$E[Wq]$ is the _____

$E[Nq]$ is the _____

The most common measure of variability in delay is the variance in Wq represented as _____ or _____

A large variance or standard deviation indicates high/low (circle on) variability in delay.

To combat high variability flight schedules incorporate slack time in their schedules. Explain how slack time addresses the issue of variability _____

Reliability is defined as the probability _____

An example of a reliability metric is the percentage of flights that arrive with 15 minutes of the scheduled arrival time.

Draw a log-normal distribution with a long right tail representing flight delays. Sketch in the region of the distribution representing the “late” flights.

Explain where the rationale for the 15 minute threshold ...

Maximum Queue Length is used by designers to determine the amount of space (e.g. taxiway lengths for departure queue) that should be provided to handle the maximum queue. This parameter is measure of the risk of the exceeding the maximum queue length.

Describe the two approaches used to compute Maximum Queue Length;

1) _____

2) _____

STOCHASTIC QUEUING BEHAVIOR

Read 23-4 and 23-6

Page 842

Stochastic delays occur when the demand rate is less than the available capacity (i.e. ρ _____ 1). This is due the _____ in the demand inter-arrival times and/or service times. These arrival clusters appear due to _____ and/or _____.

As ρ approaches 1 over a long period, the stochastic delays can be come significant.

Stochastic queuing systems are analyzed under _____ conditions.

Explain this term _____

Explain the term steady state _____

Little's Law is a description of _____

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Little's Law

Little's Law computes the following 4 parameters:

1. Total Amount of Time Spent by a User in the Queue represented by _____
2. Total Number of Users in the Queuing System, represented by _____

- 3. Waiting Time for a User, represented by W_q
- 4. Number of Users in the Queue, represented by N

W = sum of amount of time a user spends in the queue and being serviced (W_q).

N = sum of _____

Keep in mind, each of the 4 parameters are random variables. When a queuing system is in steady-state the expected values (i.e. averages) of the random variables satisfy the following relationships:

- 1. $E[N] = \lambda * E[W]$
- 2. $E[N_q] = \lambda * E[W_q]$
- 3. $E[W] = E[W_q] + (1/\mu)$

Explain each of the relationships above:

1. _____

2. _____

3. _____

Congestion vs Utilization

Page 843+

Under steady-state conditions, $E[W]$, $E[Wq]$, $E[N]$, and $E[Nq]$ increase non-linearly with respect to ρ , in proportion to the quantity $1/(1-\rho)$.

Use a spreadsheet to compute values for $1/(1-\rho)$ and sketch the relationship ρ on the x-axis) vs $1/(1-\rho)$ on the y-axis.

ρ	$1/(1-\rho)$.
0	
0.1	
0.2	
0.3	
0.4	
0.5	
0.6	
0.7	
0.8	
0.9	
1	



The actual vales for $E[W]$, $E[Wq]$, $E[N]$, and $E[Nq]$ depend on the configuration of the queuing system (i.e. number of servers) and the parameters of the queuing functions. The most common form of queuing system is an M/G/1 system. This stands for a Memoryless system with any (general) probability distribution of service times, with 1 server.

Typical M/G/1 system:

- Single server (e.g. runway)
- Demand arrives at entirely random times according to a Poisson process
- Inter-arrival times are described by a negative exponential probability distribution with parameter λ (i.e. demand rate per unit time)
- Service rate = μ

- Service time S , has variance $\sigma^2(S)$
- System has infinite queuing capacity (e.g. taxiway holds all aircraft submitted to a departure runway queue).

$$E[Wq] = \underline{\hspace{15em}}$$

$$E[Wq] = \underline{\hspace{15em}}$$

Now that you have computed $E[Wq]$ you can compute

$$E[W] = E[Wq] + (1/\mu)$$

Then you can compute,

$$E[N] = \lambda * E[W]$$

$$E[Nq] = \lambda * E[Wq]$$

Note 1: the expression for $E[Wq]$ includes the term $1/(1-\rho)$. This term is a dominant factor in the equations and determines the overall shape of the function.

Note 2: the $\sigma^2(S)$ term, the variability in Service times determines how fast the $E[.]$ grow. In general, the higher the variability in inter-arrival times (i.e. bunching or arrivals represented by λ) or the higher the variability in service times ($\sigma^2(S)$), the faster $E[.]$ increases.

Exercise #1: Study the relationship between Service Time variability and Average Number of Users in the Queue.

Use a spreadsheet to compute and chart the values for ρ vs $E[Nq]$. Assume an M/G/1 system. See table below. Explain the difference in $E[Nq]$ between system A and B.

What are the design implications of this result?

	System A $\mu = 60$ per hour $(\sigma^2(S) = 0)$		System B $\mu = 60$ per hour $(\sigma^2(S) = 0.81)$	
ρ	$E[Wq]$	$E[Nq]$	$E[Wq]$	$E[Nq]$
0				
0.1				
0.2				
0.3				
0.4				
0.5				
0.6				
0.7				
0.8				
0.9				
1				

Exercise #2: Study the relationship between Demand and Capacity.

Use a spreadsheet to compute and chart the values for ρ vs $E[Wq]$, $E[Nq]$, $E[W]$,

$E[N]$

Assume an M/G/1 system to model a departure runway during a peak departure period.

- Capacity = $\mu = 48$ per hour
- Service Times: Expected Value = 75 seconds and Standard Deviation = 25 seconds
- Demand occurs at a steady rate and can be approximated by a Poisson distribution

Note $\rho = \text{Arrival Rate}(\lambda)/\text{Capacity}(\mu)$

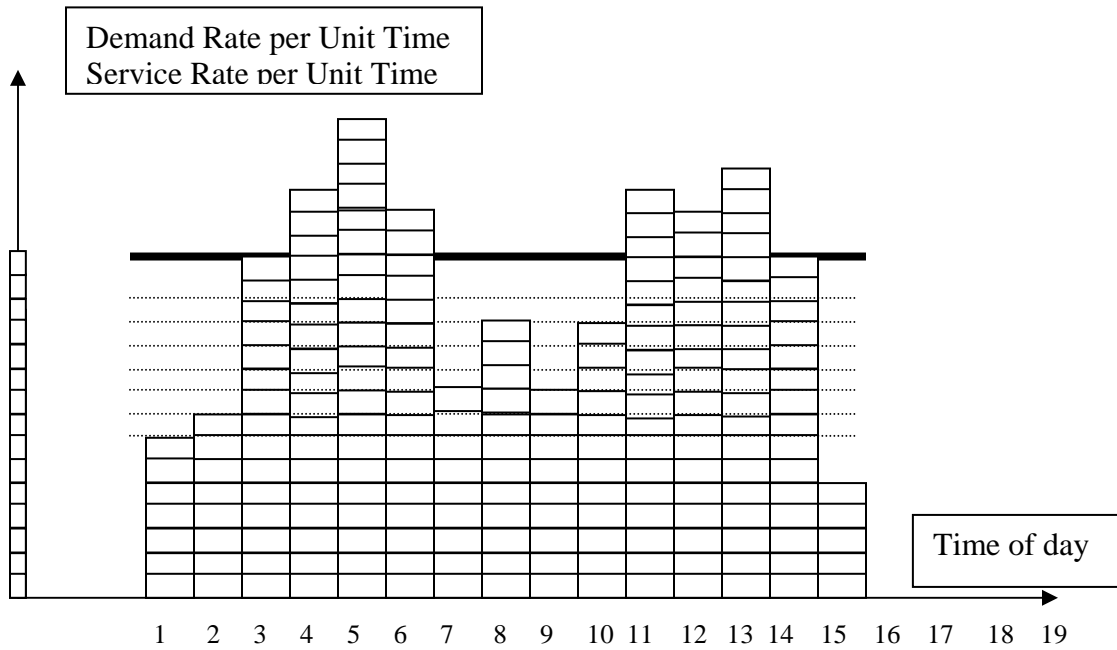
		System A
--	--	-----------------

ρ	$\lambda = \rho/\mu$	$E[Wq]$	$E[Nq]$	$E[W]$	$E[N]$
0.6					
0.7					
0.8					
0.9					
1					

- a) Explain what happens as demand approaches capacity?
- b) What is the impact on variability as demand approaches capacity?
- c) What are the implications for the regulating schedules at airports?

REVIEW:

OVERLOADED QUEUING SYSTEMS



Identify the following parameters. Show all work:

1 Daily Total Capacity of the Arrival Runway (i.e. service rate)

2 Arrival Rate (i.e. demand rate for period)

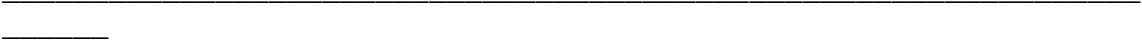
3 Daily Total Demand for the Arrival Runway

4 Time periods with Demand in excess of Capacity

5 Time periods in which flights will be delayed

6 Daily Total Demand over Capacity Ratio

7 Do you expect to delays to occur? Why



Learning Objectives:

1. Understand the dynamics of delays that result from over-scheduled resources
2. Understand the impact of “exemptions” to the dynamics of delays that result from over-scheduled resources
3. Understand the impact of “cancellations” to the dynamics of delays that result from over-scheduled resources
4. Understand the impact of “cancellations with compression” to the dynamics of delays that result from over-scheduled resources
5. Understand the impact of “slot swapping” to the dynamics of delays that result from over-scheduled resources
6. Understand the implications of over-scheduled resources on Proportional Equity
7. Understand the canonical form of the equations for overscheduled resources

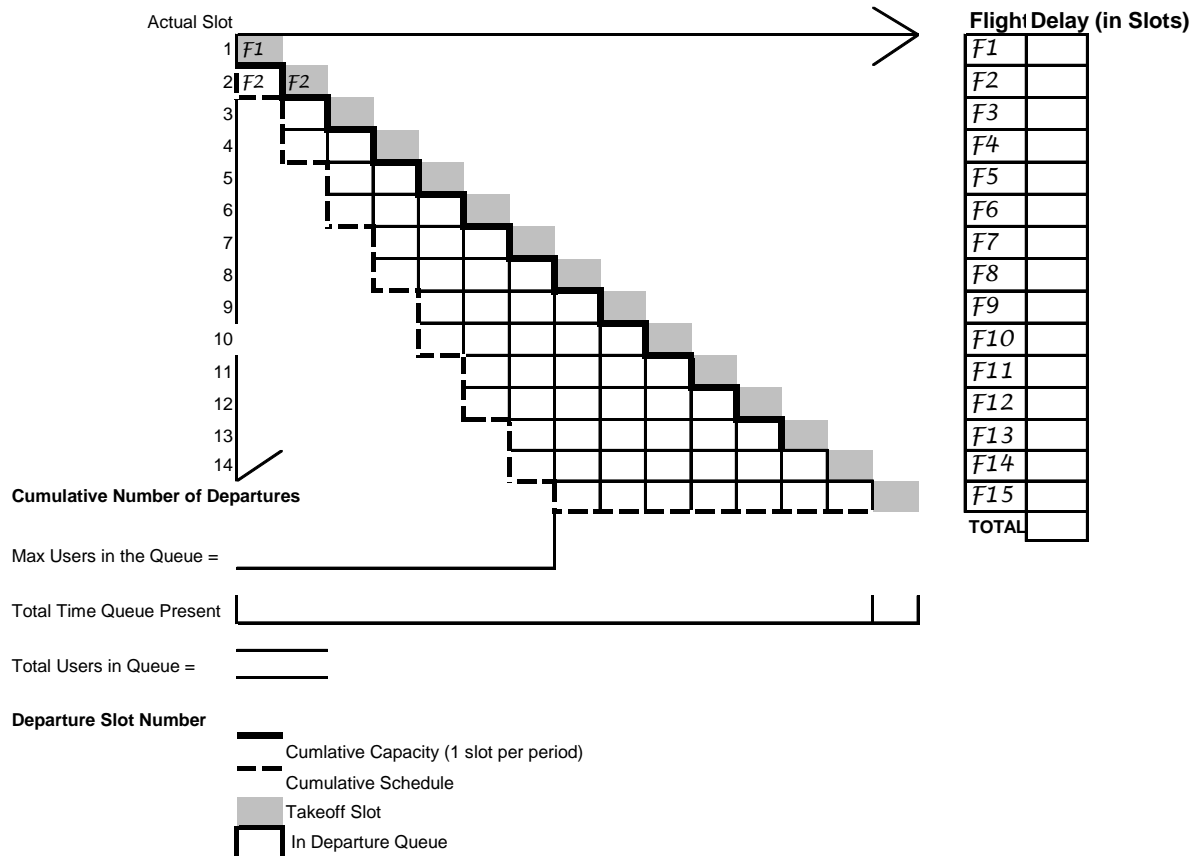
Baseline Overscheduled Queueing Dynamics

15 flights are scheduled into 8 slots. Each slot can only service one flight at a time.

1. Complete the Queueing matrix below. Insert each flight into the queue in the appropriate slot. Use the partial matrix in the middle of the diagram.

CUMULATIVE DETERMINISTIC QUEUEING DIAGRAMS

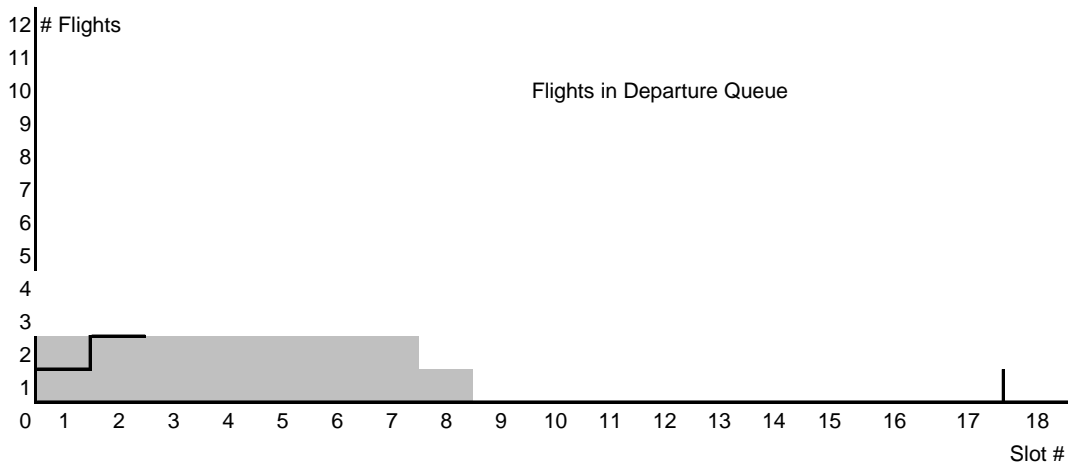
Departure Slots	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Schedule (AirlineFlight#)	F1	F3	F5	F7	F9	F11	F13	F15	-	-	-	-	-	-	-
	F2	F4	F6	F8	F10	F12	F14								



Total Delay Time = Total Flights in Departure Queue
 = Area between Cumulative Capacity
 and Cumulative Schedule

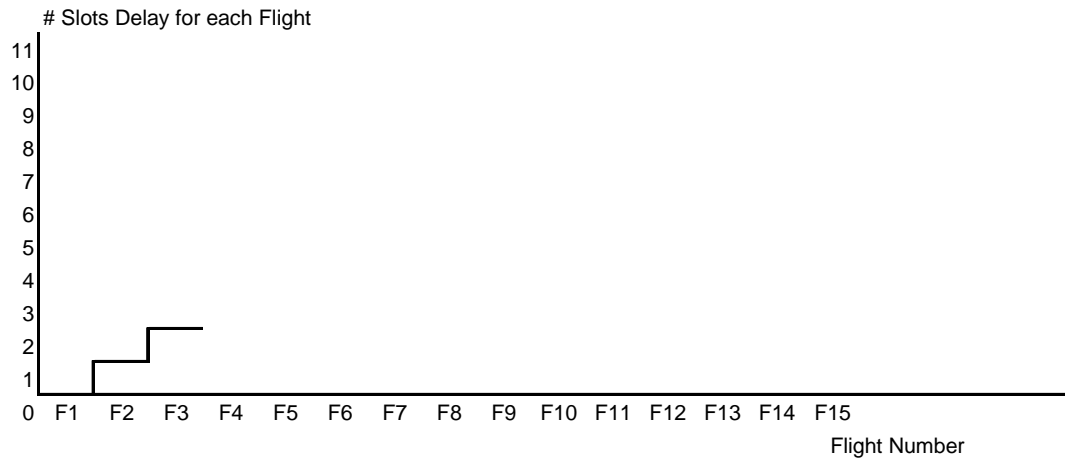
2. Compute Individual Flight and Total Flight delays. Use the table on the right hand-side.
3. Identify the Max Users in the Queue
4. Identify the Total Time the Queue is present

5. Identify the Total Number of Users in the Queue
6. Complete the chart below identifying the number of Flights in the queue for each slot.
 - a. Are flights in the non-overscheduled slots affected by overscheduling in the earlier slots
 - b. At which slot does the queueing peak



7. Complete the chart below describing the Delay experienced by each individual flight.
 - a. Is flight delay allocated equally for each flight? _____
 - b. Which flights receive the least delays? _____
 - c. Which flights receive the most delays? _____
 - d. How much delay does the flight in the non-overscheduled slot receive? Is _____

Delay Analysis Workbook



Overscheduled Queueing Dynamics – Exempted Flights

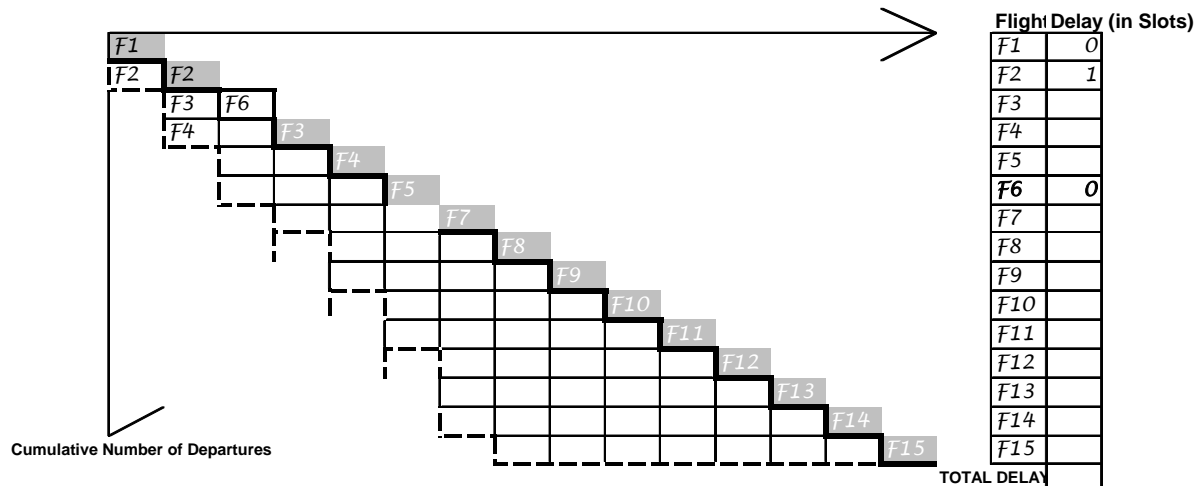
15 flights are scheduled into 8 slots. Each slot can only service one flight at a time.

Flight F6 is “exempted” from queueing. Exemptions occur for international flights operating under bi-lateral access agreements, for “life-guard” flights carrying donor organs, flights that are not within the scope of the delay allocation rules (e.g. long distance flights), flights that may have already been allocated (long) delays.

Exempted flights do not queue. They are given their originally scheduled slots (or a preferential slot).

- Complete the Queueing matrix below. Insert each flight into the queue in the appropriate slot. Use the partial matrix in the middle of the diagram.

Departure Slot	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Schedule (Air)	F1	F3	F5	F7	F9	F11	F13	F15	-	-	-	-	-	-	-
	F2	F4	F6	F8	F10	F12	F14								



Max Users in the Queue = _____

Total Time Queue Present _____

Total Users in Queue = _____

Departure Slot Number

- Cumulative Capacity (1 slot per period)
- - - Cumulative Schedule
- █ Takeoff Slot
- In Departure Queue

Total Delay Time = Total Flights in Departure Queue
 = Area between Cumulative Capacity
 and Cumulative Schedule

2. Compute Individual Flight and Total Flight delays. Use the table on the right hand-side. How do Total Flight Delays compare with the Baseline queueing? Explain.

3. Identify the Max Users in the Queue. Compare to Baseline queueing? Explain.

4. Identify the Total Time the Queue is present. Compare to Baseline queueing? Explain.

5. Identify the Total Number of Users in the Queue. Compare to Baseline queueing? Explain.

Overscheduled Queueing Dynamics – Cancelled Flights

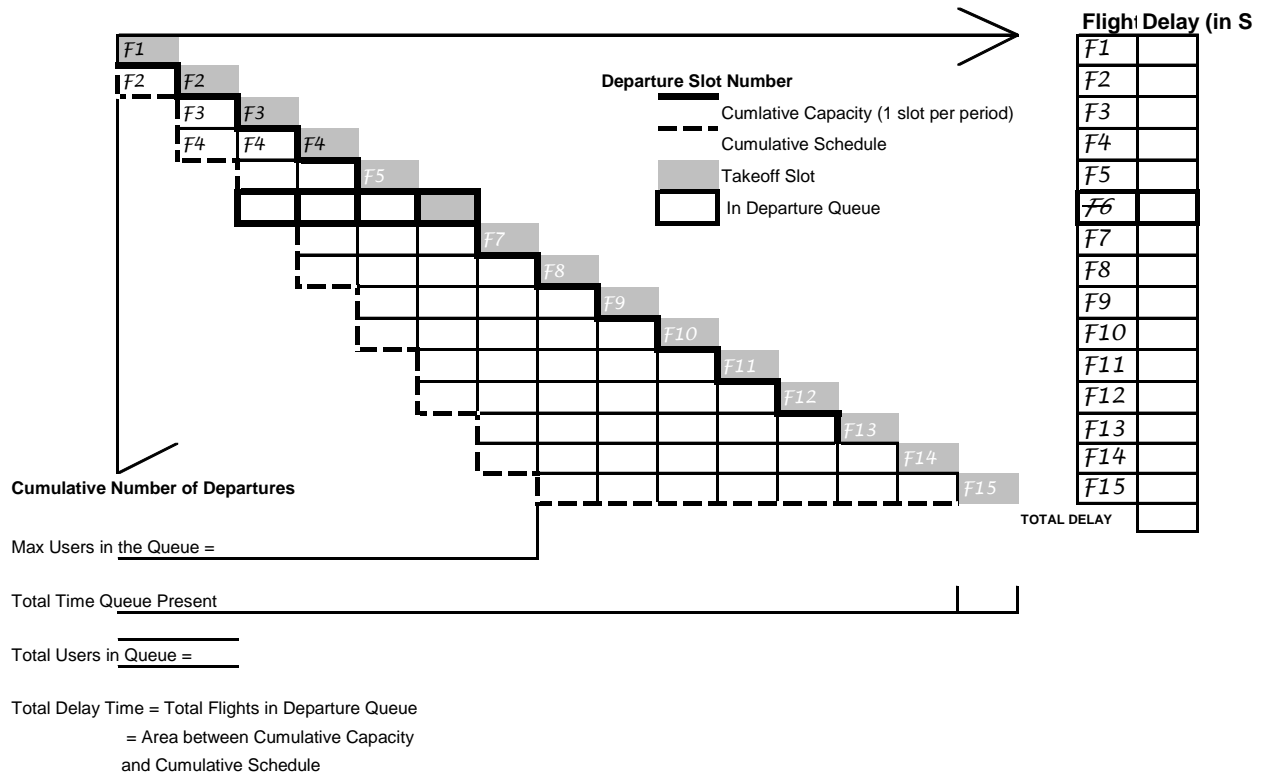
15 flights are scheduled into 8 slots. Each slot can only service one flight at a time.

Flight F6 is “cancelled”. Cancelled flights occur when the flight is unsafe to operate (e.g. mechanical failure), or when the airline “tactically” cancels a flight to save money, a flight-crew is not available, or to re-align aircraft to maintain the integrity of it’s network.

Cancelled flights are not included in the queue, but their slot remains unused.

1. Complete the Queueing matrix below. Insert each flight into the queue in the appropriate slot. Use the partial matrix in the middle of the diagram.

Departure Slot	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Schedule (Air)	F1	F3	F5	F7	F9	F11	F13	F15	-	-	-	-	-	-	-
	F2	F4	F6	F8	F10	F12	F14								



2. Compute Individual Flight and Total Flight delays. Use the table on the right hand-side. How do Total Flight Delays compare with the Baseline queueing? Explain.

3. Identify the Max Users in the Queue. Compare to Baseline queueing? Explain.

4. Identify the Total Time the Queue is present. Compare to Baseline queueing? Explain.

5. Identify the Total Number of Users in the Queue. Compare to Baseline queueing? Explain.

Overscheduled Queueing Dynamics – Cancelled Flights with Compression

15 flights are scheduled into 8 slots. Each slot can only service one flight at a time.

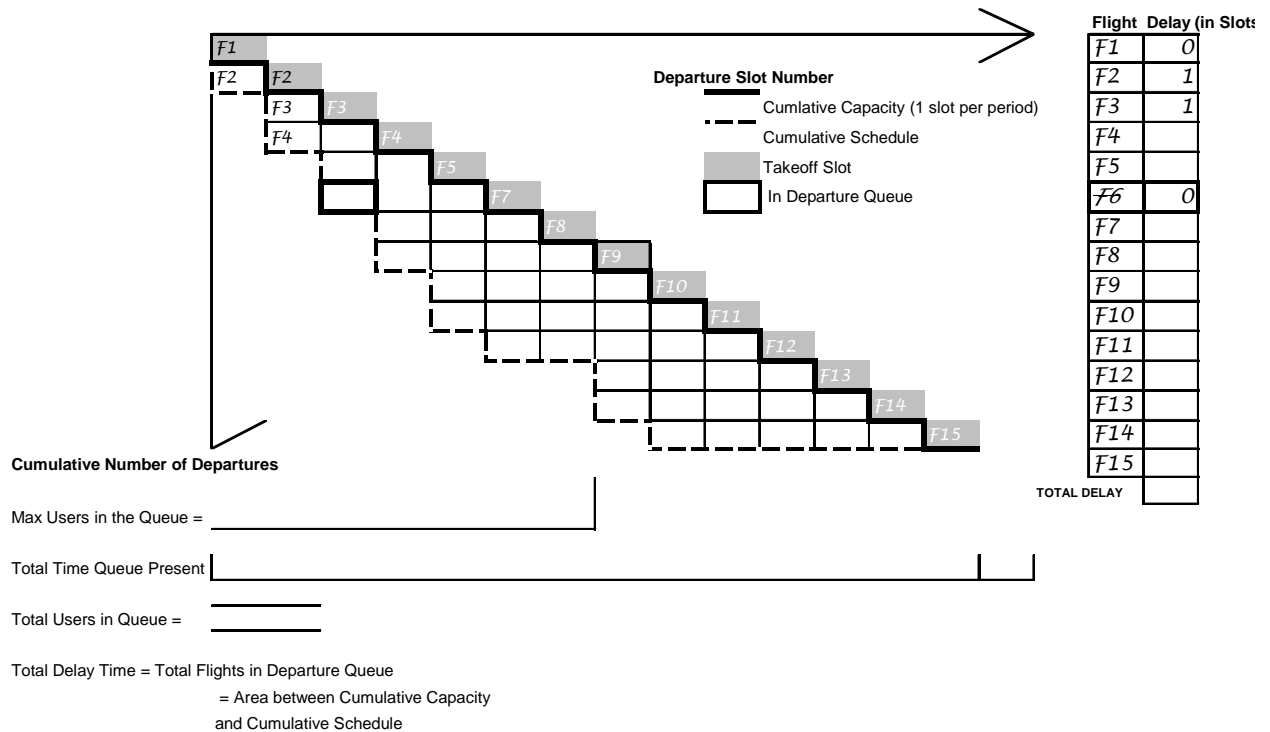
Flight F6 is “cancelled”. This time rather than leave the slot unused, the airline that operates Flight 6 gives it’s allocated slot away. The subsequent flights are then “compressed” – each subsequent flight moves up to fill any gaps.

Compression is when unused slots are filled by moving all downstream flights up to fill the gaps.

Note: Compression can only take place, when the owner of the unused slot makes this information available.

1. Complete the Queueing matrix below. Insert each flight into the queue in the appropriate slot. Use the partial matrix in the middle of the diagram.

Departure Slots	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Schedule (AirlineFlight#)	F1	F3	F5	F7	F9	F11	F13	F15	-	-	-	-	-	-	-
	F2	F4	F6	F8	F10	F12	F14								



2. Compute Individual Flight and Total Flight delays. Use the table on the right hand-side. How do Total Flight Delays compare with the Baseline queueing? Explain.

3. Identify the Max Users in the Queue. Compare to Baseline queueing? Explain.

4. Identify the Total Time the Queue is present. Compare to Baseline queueing? Explain.

5. Identify the Total Number of Users in the Queue. Compare to Baseline queueing? Explain.

6. Under what conditions would the operator of flight F6 gain by giving up their unused slot? Explain.

Overscheduled Queueing Dynamics – Cancelled Flights with Slot Swapping

15 flights are scheduled into 8 slots. Each slot can only service one flight at a time.

Airline B operates Flights: F2, F4, F6, F8, F10, F12, F14. Flight F6 is cancelled. F8 is moved in F6 slot, F10 is moved into F8 slot ...

The rules for slot swapping: Each airline may freely substitute flights within the set of it's own flights slots as long as those flights are not moved earlier than their originally scheduled slot.

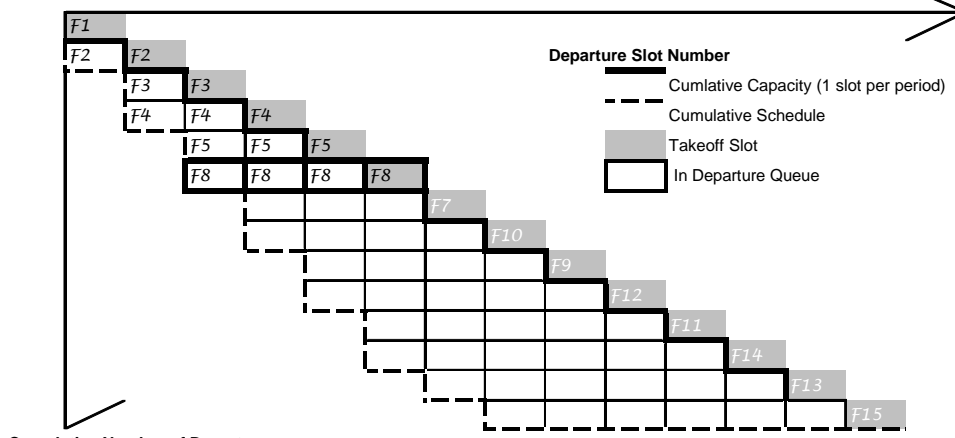
Note: This rule is particularly useful for airlines that cancel flights early in the over-scheduled period.

- Complete the Queueing matrix below. Insert each flight into the queue in the appropriate slot. Use the partial matrix in the middle of the diagram.

Departu	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Schedu	F1	F3	F5	F7	F9	F11	F13	F15	-	-	-	-	-	-	-
	F2	F4	F6	F8	F10	F12	F14								

Airline B operates Flights (F2, F4, F6, F8, F10, F12, F14). Flight F6 is cancelled. F8 is moved in F6 slot, F10 is moved into F8 slot...

Departu	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Schedu	F1	F3	F5	F7	F9	F11	F13	F15	-	-	-	-	-	-	-
	F2	F4	F8	F10	F12	F14									



Flight Delay (in Slots)

F1	
F2	
F3	
F4	
F5	
F6	
F7	
F8	
F9	
F10	
F11	
F12	
F13	
F14	
F15	
TOTAL DELAY	

Max Users in the Queue =

Total Time Queue Present

Total Users in Queue =

Total Delay Time = Total Flights in Departure Queue
= Area between Cumulative Capacity
and Cumulative Schedule

2. Compute Individual Flight and Total Flight delays. Use the table on the right hand-side. How do Total Flight Delays compare with the Baseline queueing? Explain.

3. Identify the Max Users in the Queue. Compare to Baseline queueing? Explain.

4. Identify the Total Time the Queue is present. Compare to Baseline queueing? Explain.

5. Identify the Total Number of Users in the Queue. Compare to Baseline queueing? Explain.

6. Under what conditions would the operator of flight F6 gain by giving up their unused slot? Explain.

Equity

Every society has rules for sharing goods and burdens among its members (Young, 1994). Some resources are managed through property rights and liabilities that are held and traded by private individuals or held by enterprises according to complex financial regulations. Other property rights are held by a governing entity and allocated according to societal needs. The mechanism for the distribution of property rights expresses the notions of equity in the division of the resources deemed reasonable by societal norms. The appropriateness of the equity is determined in part by principle and in part by precedent.

There are three main decisions that must be made in the allocation of commonly held property: (1) the *supply decision* determines the amount of resources to be distributed (e.g. arrival slots). (2) the *distributive decision* determines the principles and methods used to allocate the resources (e.g. first-scheduled/first-served), and (3) the *reactive decision*: determines the owners or users to the allocation scheme (e.g. slot substitutions and slot swapping). The focus of this paper is the implications of the distributive decision.

Varieties of Equity

Equitable allocation should be a simple process. Each party is allocated an equal distribution measured according to a single yardstick. The reality is that allocated resources are not equal. Claimant parties are in different situations and the agreement of single yardstick is difficult to achieve (Rae 1981). A wide range of philosophers (e.g. Aristotle, Maimonides) have examined the “combinatorics” of allocation of asymmetric resources to claimants using various yardsticks. These philosophers have developed appropriate allocation schemes for specific combinations. One of the emergent themes of these allocation schemes is that the equity formulas are usually based, either explicitly or implicitly, on a standard of comparison that ranks the various claimants on their relative desert (Young, 1994; pg 80).

Proportionality and Proportional Equity

One of the oldest and most widely used distributive principles is one that ranks claimants rights. This is the “Principle of Proportionality.” Proportionality is implicit in the mechanism of First-Come/First-Serve used in Air Traffic Control (ATC) and is the explicit in the mechanism of First-Scheduled/First-Served used in Traffic Flow Management (ATFM). This principle allocates the resources in proportion to the demand for the resource such that groups (e.g. airlines, passengers with specific demographics, or flights with specific emissive properties) will receive delays in proportion to their number of flights scheduled.

Proportional Equity is defined by the following equation:

Proportional Equity for Group (i) =

$$\frac{[\text{Total Delay for Group (i)} / \text{Total Delay for all Groups}]}{[\text{Number of Flights for Group (i)} / \text{Total Flights for all Groups}]}$$

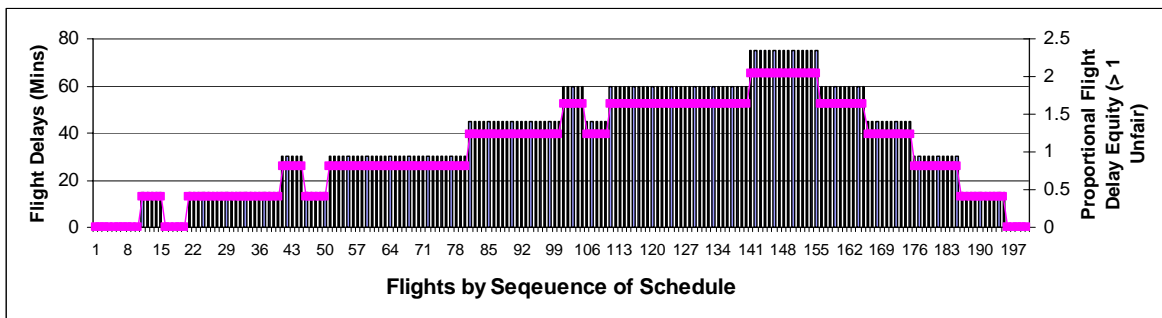
where i are groups of users (e.g. airlines) 1 through n.

The numerator represents the proportion of delays allocated to Group (i) with respect to the total delays allocated to all the Groups. The denominator represents the proportion of flights flown by Group (i).

When the proportion of delays is equivalent to the proportion of flights, the equity for the group is equal to 1. Proportional equity less than 1, implies that the group was allocated delays proportionately less than the number of flights scheduled. This group benefited from the allocation process. Proportional equity greater than 1, implies that the group was allocated delays proportionately more than the number of flights scheduled. This group was penalized by the allocation process.

Proportional Equity for Individual Flights

The equity of individual flights is entirely based on the magnitude of the spill-over of preceding flights. As a consequence, flights scheduled early in the congested period receive less than the average delay and have a proportional equity of less than 1. Flights scheduled late in the congested period, receive more delays and experience an equity of greater than 1. These results are illustrated in Figure 5. The vertical bars represent the delays allocated to individual flights (left y-axis). The magenta line defines the proportional equity for each flight (right y-axis).



Proportional equity for individual flights is asymmetric. Flights are shown from first scheduled to last scheduled. The left y-axis identifies the delays experienced by each flight. The right y-axis identifies the proportional equity of each flight. Flights with proportional equity less than one experience an allocation advantage. Flights with proportional equity greater than one experience an allocation disadvantage.

Figure 5.

Proportional Equity for Groups of Flights

Flights are scheduled by the individual airlines to meet the airlines network and connecting passenger and crew objectives and are constrained by the available airline resources. When flights are scheduled by independent airlines, without a *priori* knowledge of the schedules of other airlines, over-scheduling is feasible. Further, the independent scheduling results in an aggregate schedule that does not follow any rigid structure that could lead to symmetry in allocation of flight delays or equity. This schedule is effectively generated by a random process. Groupings of scheduled flights will experience proportional equity relative to the position of the flights in the schedule. A group of flights disproportionately positioned in the front (or back) of the congested period will experience less (or more) proportional equity.

Exercise:

Assume Flights are operated by airlines as follow:

Airline A operates Flights: F1, F7, F9, F11, F13, F15

Airline B operates Flights: F2, F4, F6, F8, F10, F12, F14.

Airline C operates Flights: F3, F5

Compute the Proportional Equity for each Airline and

		Baselin e	Exemptio n	Cancelle d	Cancelled with Compressio n	Cancelle d with Swappin g
	Total Delays					
	Total Number of Flights					
Airlin e A	Total Delays					
	Number of Flights					
	Proportio nal Equity					
Airlin e B	Total Delays					
	Number of Flights					
	Proportion					

	al Equity					
Airlin e C	Total Delays					
	Number of Flights					
	Proportion al Equity					

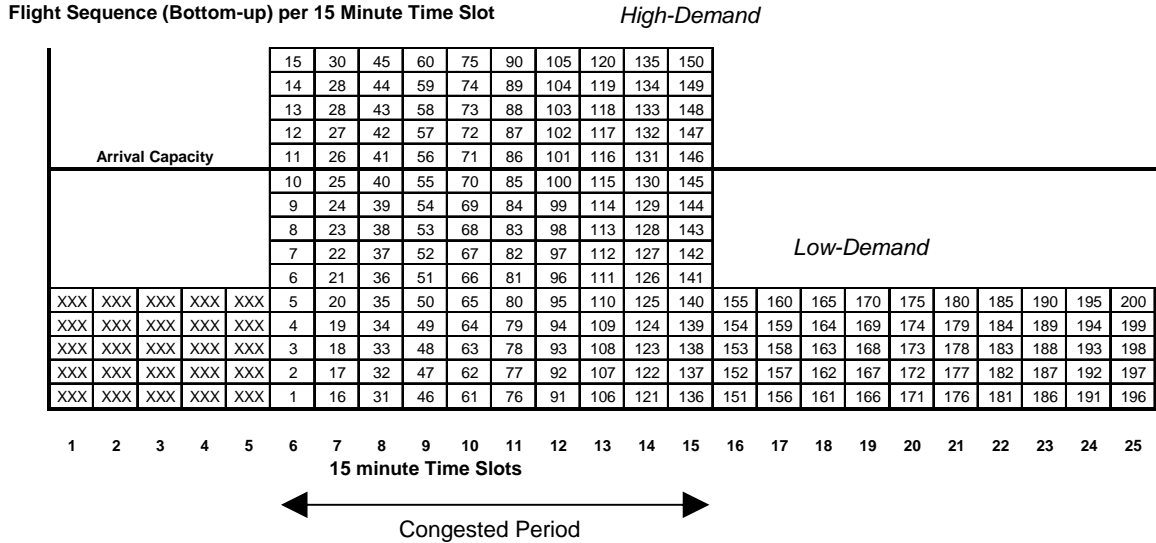
1. Under which queueing scenario is Inter-airline Proportional Equity best (i.e. Airline Proportional Equity closest to 1)? Explain.

2. What role does position in the schedule have on Inter-airline Proportional Equity ? Explain.

3. How does Cancellations, Cancellations with Compression, and Slot Swapping affect Inter-airline Proportional Equity? Explain.

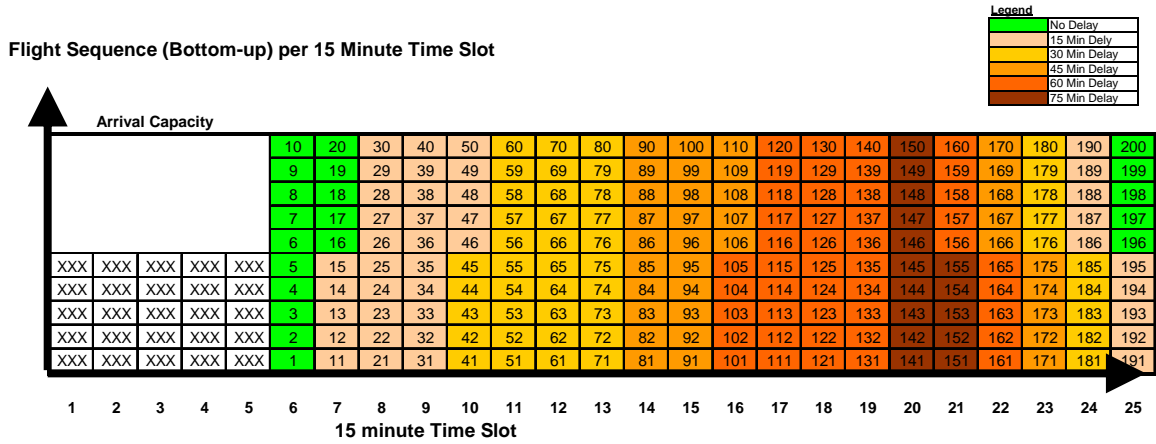
GENERALIZED MODEL OF OVERSCHEDULED QUEUEING DELAYS

A canonical representation of the scheduled over-utilization of a scarce resource is illustrated in Figure 1. In this scenario, flights are scheduled in 15 minutes periods. Starting at 15 minute period #6, flights are scheduled in excess of the *available capacity* for a period known as the congested period. Following the congested period, the *low-demand* provides a reservoir to absorb spill-over delays. The flights are numbered in the order in which they are scheduled.



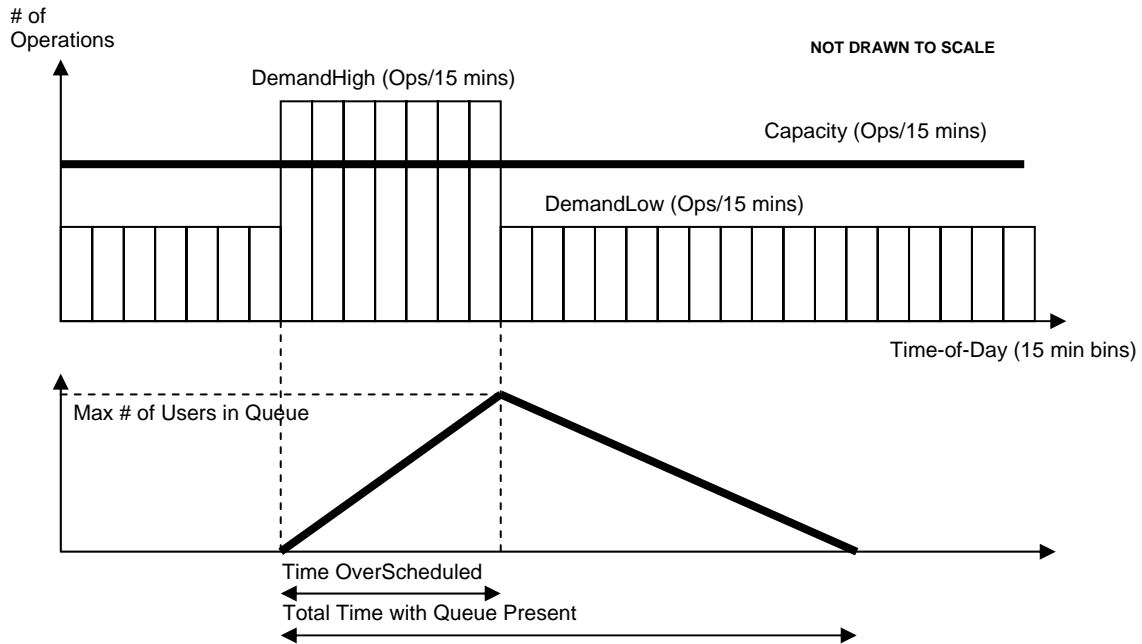
Scheduled over-utilization of a scarce resource.
Figure 1

The systemic allocation of delays to individual flights according to the order of scheduling is illustrated in Figure 2. The color-code identifies the degree of delay experienced by each flight. Flights in excess of the capacity in the first 15 minute period, spill-over into the second 15 minute period escalating the delays assigned to these flights. The spill-over cascades through the congested period (15 minute time slots 6 through 15) and into the period following the congested period (15 minute time slots 16 through 25).



Flights allocated slots within the available capacity. Note that the effect of scheduled over-utilization spills-over resulting in increasing delays to flights later in the schedule. Delays also spill-over into period after the congested period.
Figure 2.

The sum of the individual flight delays, known as the Total Delay Time, is a function of the degree of over-scheduling as well as the reservoir of capacity following the congested period. As shown in Figure 3, a queue of flights builds until the end of the congested period, at which time the reservoir of capacity allows the queue to dissipate.



Queuing of flights due to over-scheduling. The Total Delay time is the area under the Queueing function.
Figure 3

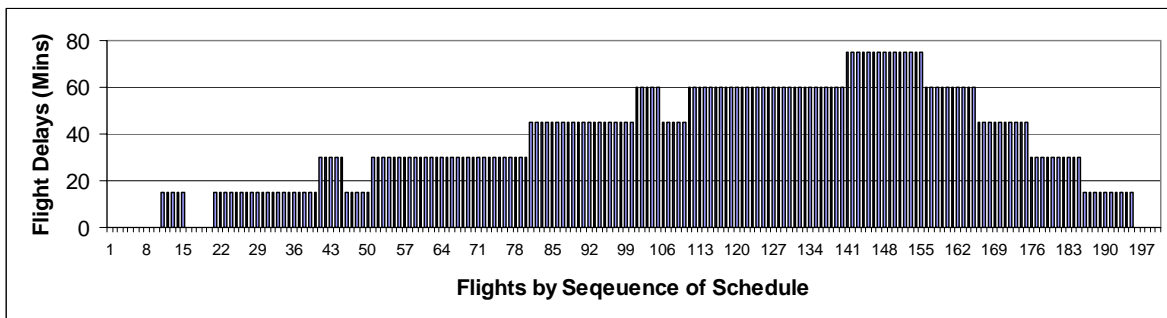
The Total Delay Time resulting from the scheduled utilization is defined by the equation:

$$\text{Total Delay Time} = \frac{1}{2} * (\text{Duration of Congested Period})^2 * (\text{HighDemand} - \text{Capacity}) * \left[1 + \frac{\text{HighDemand} - \text{Capacity}}{\text{Capacity} - \text{LowDemand}} \right]$$

This equation is derived by calculating the area under the queue curve in Figure 3. The equation highlights the following properties:

- (1) Conservation of Total Delays. The Total Delay is independent of the order of the flights. The Total Delay is dependent on the relationship amongst the four terms: Capacity, High_Demand, Low_Demand and Congested_Period. Saying this another way, the only way to reduce the Total Delay is to remove flights.
- (2) Duration of Congested Period is Critical The factor in the equation that has the biggest effect is the duration of the congested period ($T_{\text{CongestedPeriod}}$). This term is squared. For every additional unit of time in the congested period, the Total Delays increase geometrically.
- (3) Reservoirs are Critical: The Total Delays is not only dependent on the degree of over-scheduling, but also on the degree of under-scheduling after the congested period. The degree of under-scheduling provides a reservoir to absorb the spill-over from the congested period. A low degree of under-scheduling can result in extending the queue significantly.

The delays experienced by individual flights are shown in Figure 4. Flights early in the congested period experience relatively low delays. Flights at the end of the congested period and flights right after the congested period experience the highest delays.



*Delays experienced by individual flights
Figure 4.*

- (4) Asymmetry of Individual Flight Delays: The delays assigned to individual flights are a function of the location of the flight in the schedule. Flights scheduled early in the congested period, are allocated less delays than those flights later in the congested period.

$$\text{Max Users in Queue} = \frac{(\text{DemandHigh} - \text{Capacity}) * \text{TOverScheduled}}{\text{Capacity} - \text{DemandLow}}$$

Determined by:

- Degree of Over-scheduling
- Duration of Over-scheduling

Total Time Queue Present =

$$\text{TOverScheduled} + \frac{(\text{DemandHigh} - \text{Capacity}) * \text{TOverScheduled}}{(\text{Capacity} - \text{DemandLow})}$$

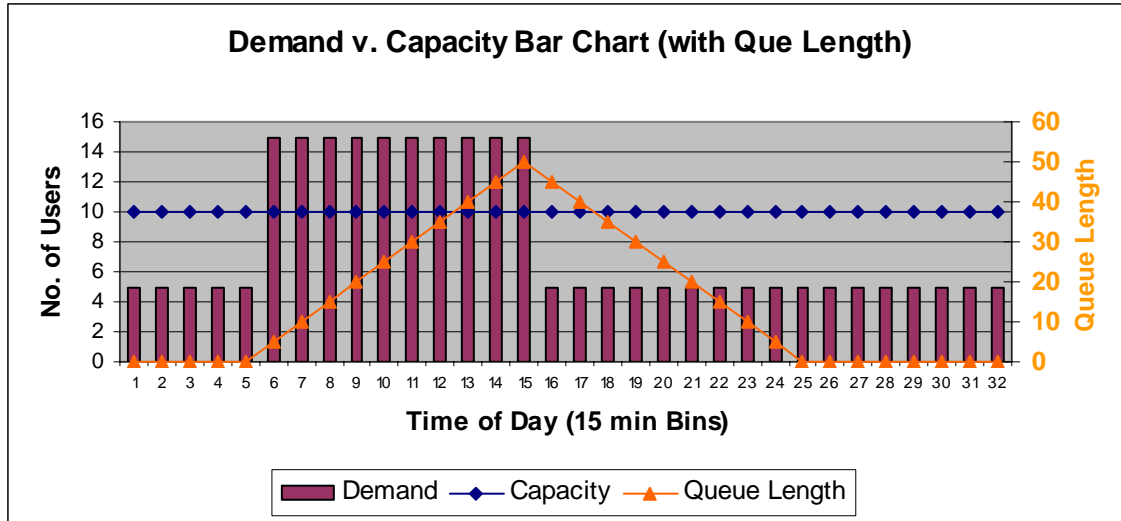
Spill-over effect depends on degree of over scheduling and available capacity *after* the over-scheduled slots

Total Users in Queue =

//Users in the Overscheduled region minus the 1st batch that do not queue
 [TOverScheduled * (DemandHigh)] - [1 * Capacity] +

//Users in the duration of the queue
 [$\frac{(\text{DemandHigh} - \text{Capacity}) * \text{TOverScheduled}}{(\text{Capacity} - \text{DemandLow})}$] * DemandLow

Exercise:



Capacity = 10 flights in 15 minute period
 HighDemand = 15 flights in 15 minute period
 LowDemand = 5 flights in 15 minute period
 Congested Period = T = 10 15 minute periods.

Compute the following measures. **Show all work.**

1. Max Users in the Queue
2. Total Time Queue Present
3. Total Delay Time
4. Total Users in the Queue
5. Expected Delay for Users in the Queue

Answer Key:

Max Users in Queue 50
Total Time with Queue Present 20
Total Delay Time 500
Total Users in the Queue 190
Expected Delay for Users in Queue 2.63157895

10 Capacity
5 Demand Low
15 Demand High
10 Congested Period