Simulating Aircraft Dynamics (i.e. Taxi) on the Airport Surface

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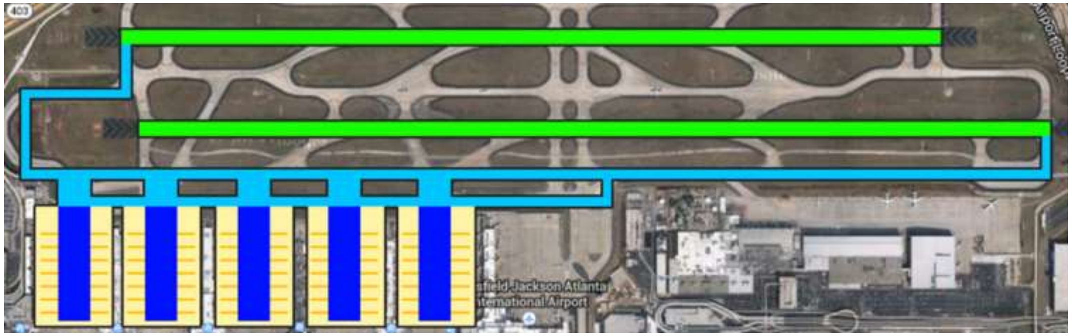
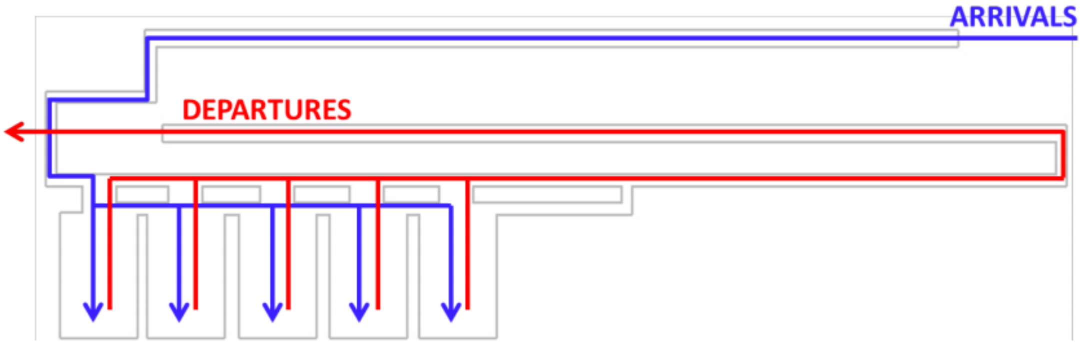
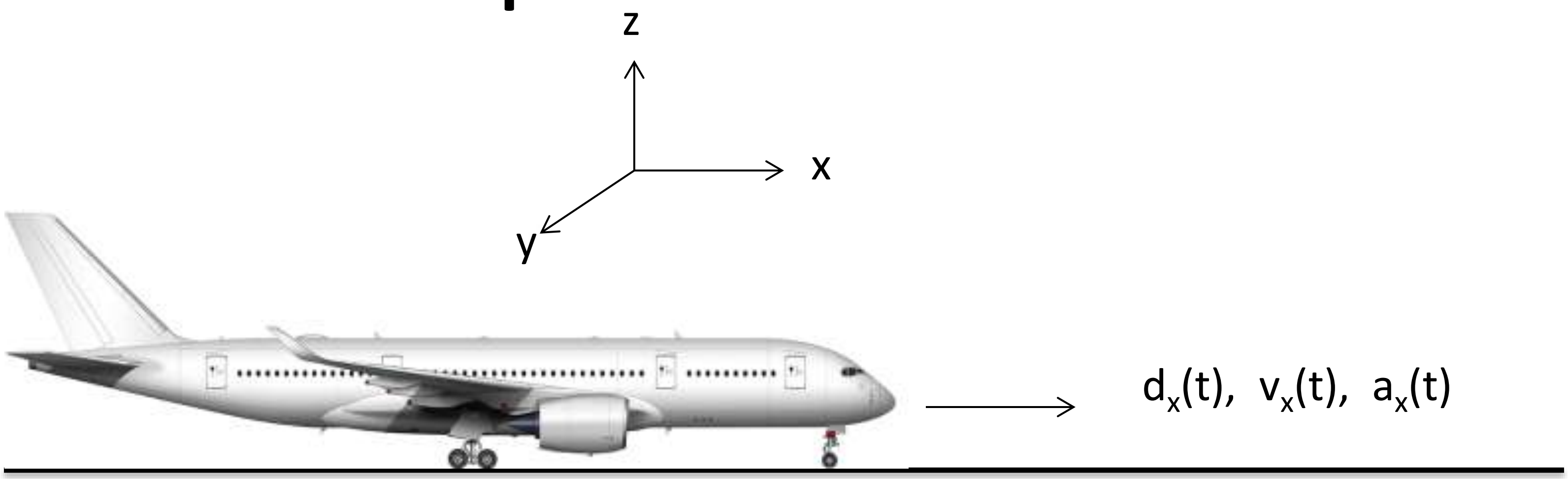
SYST 460/560

Simulating Aircraft Movement on the Airport Surface



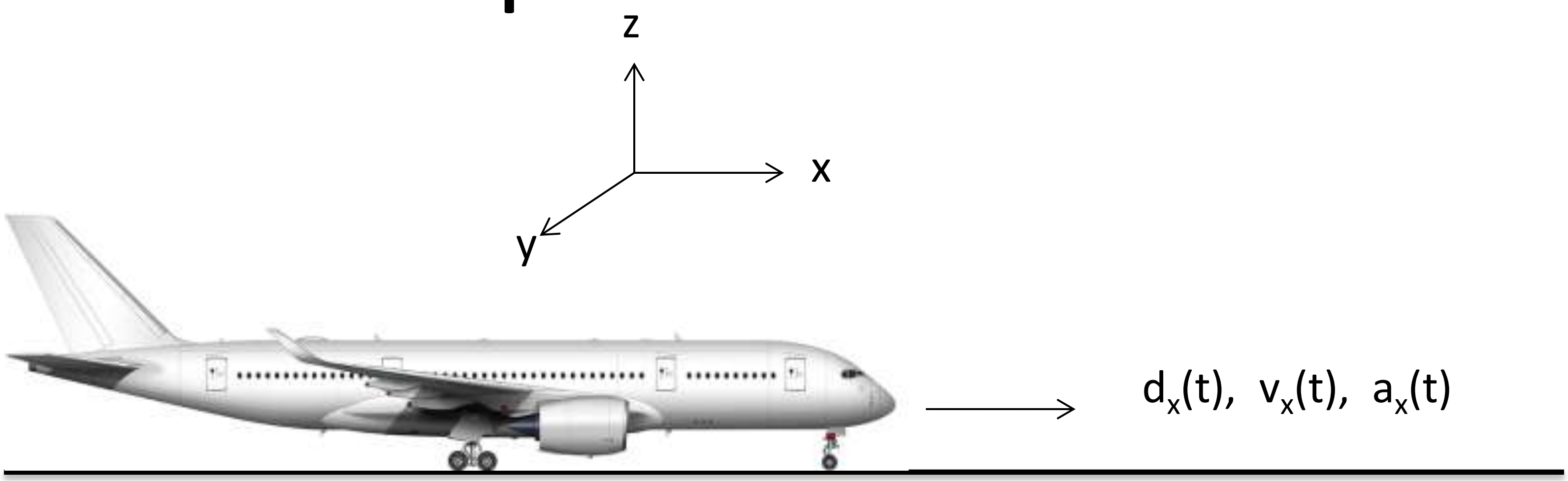


Equations of Motion



d_x = Distance covered in x direction
v_x = Velocity in x direction
a_x = Acceleration in x direction

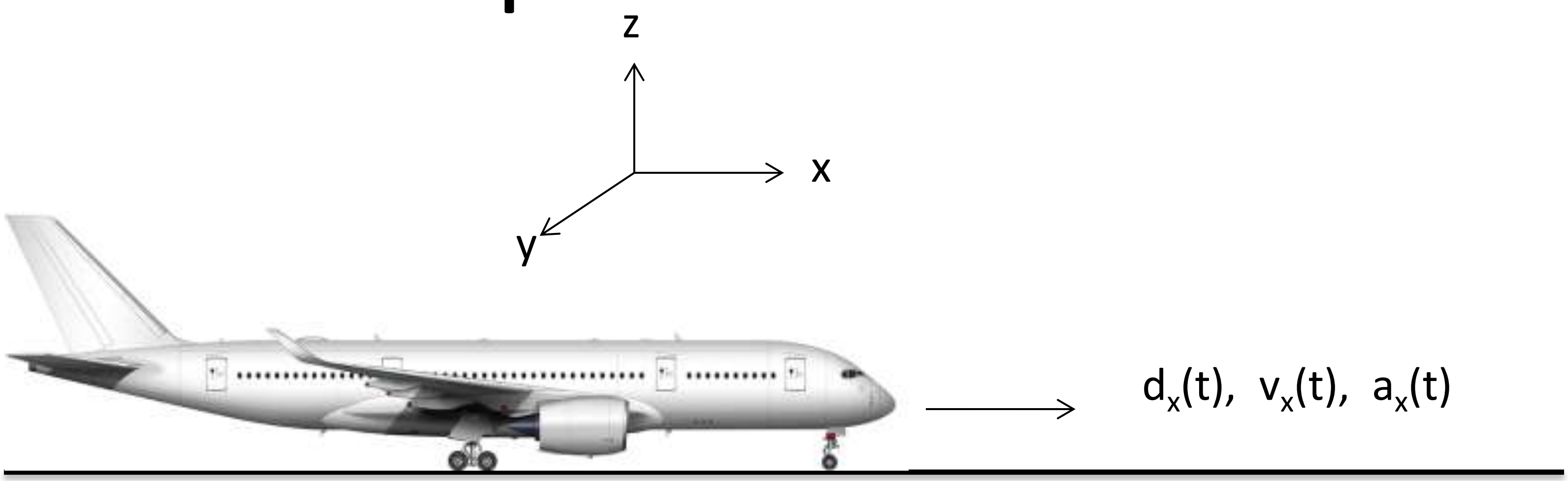
Equations of Motion



$d_x(t)$ = Distance covered in x direction
$v_x(t)$ = Velocity in x direction
$a_x(t)$ = Acceleration in x direction

<u>Relationship between parameters</u>:
$d_x(t) = \underline{v_x} * \text{Time}$
$v_x(t) = \underline{a_x} * \text{Time} \quad = d_x / \text{Time}$
$a_x(t) = \underline{v_x} / \text{Time}$

Equations of Motion



$d_x(t)$ = Distance covered in x direction
$v_x(t)$ = Velocity in x direction
$a_x(t)$ = Acceleration in x direction

<u>Simulation Equations</u>:

$a_x(t)$ = Input value

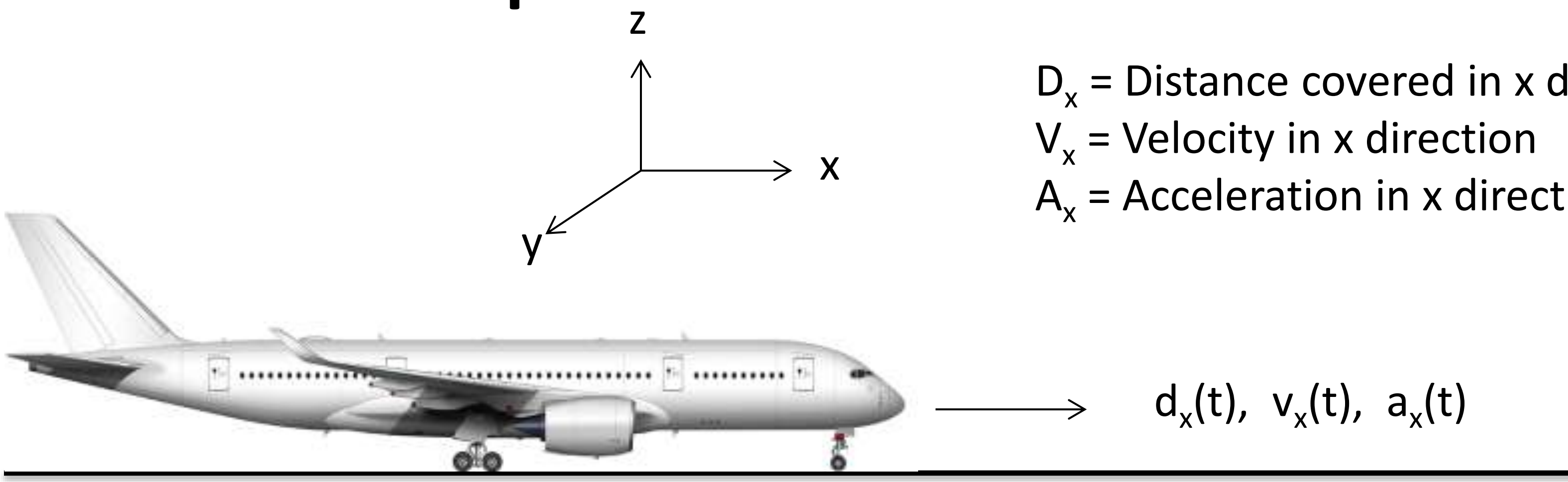
$v_x(t) = (a_x(t) * \Delta T) + v_x(t-1)$

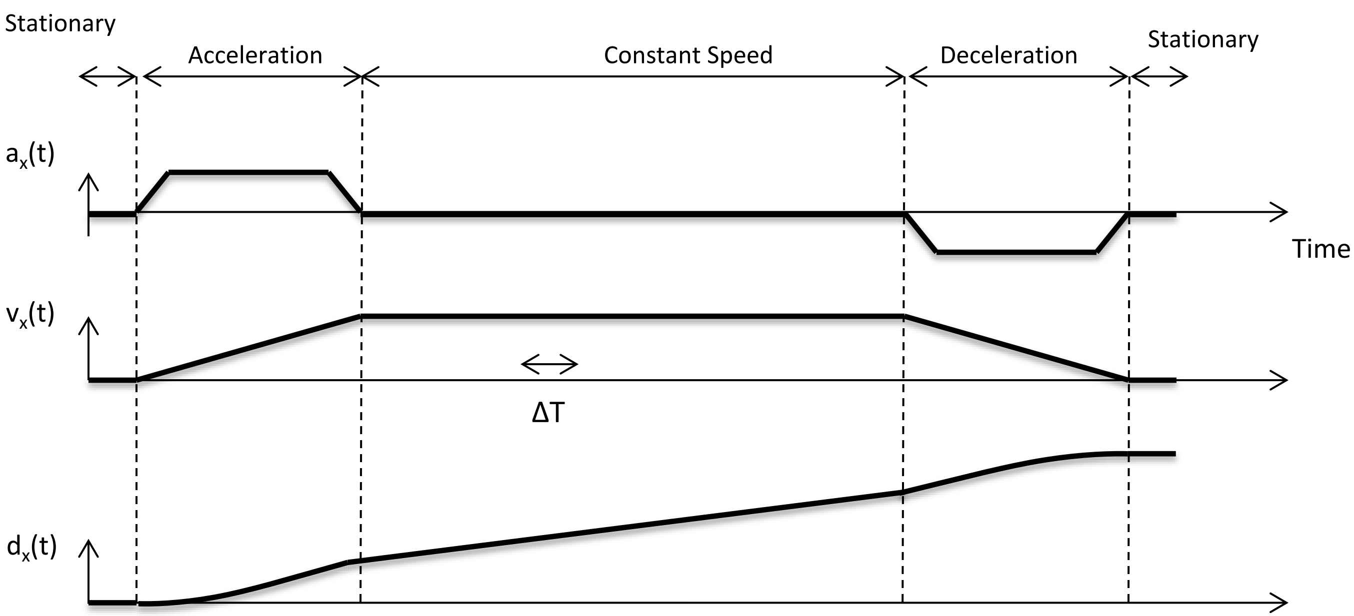
$d_x(t) = (v_x(t) * \Delta T) + d_x(t-1)$

$\Delta T = (t) - (t-1)$

Equations of Motion

D_x = Distance covered in x direction
V_x = Velocity in x direction
A_x = Acceleration in x direction





Simulation Equations:

$a_x(t)$ = Input values (t)

$v_x(t) = (a_x(t) * \Delta T) + v_x(t-1)$

$\Delta T = (t) - (t-1)$

$d_x(t) = (v_x(t) * \Delta T) + d_x(t-1)$

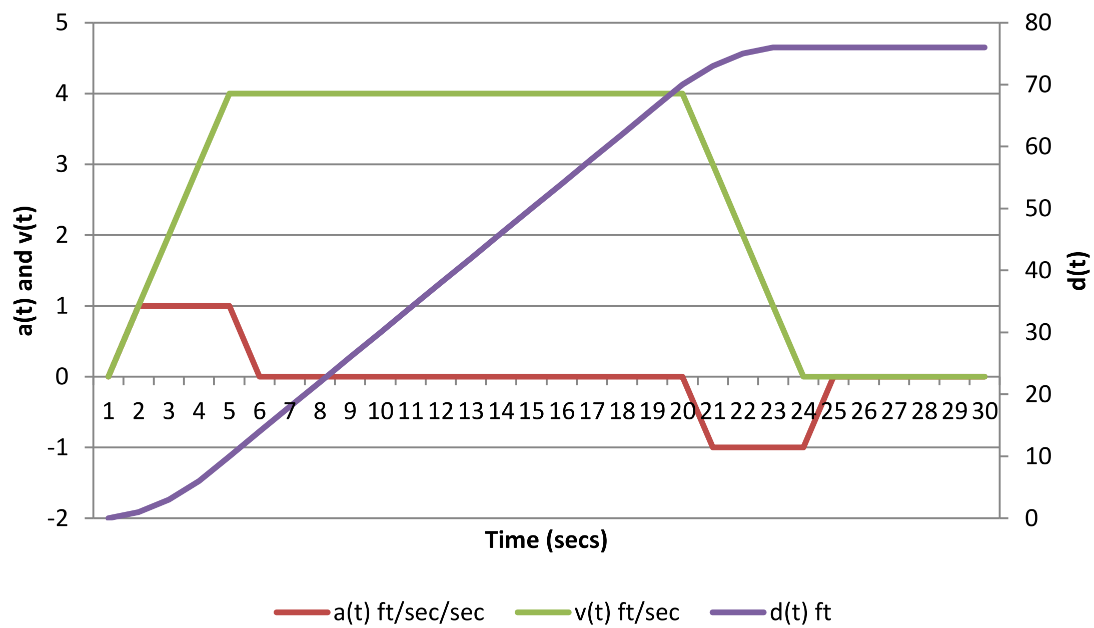
Simulation

INPUT

OUPUTS

T (secs)	a(t) ft/sec/sec	v(t) ft/sec	d(t) ft
1	0	0	0
2	1	1	1
3	1	2	3
4	1	3	6
5	1	4	10
6	0	4	14
7	0	4	18
8	0	4	22
9	0	4	26
10	0	4	30
11	0	4	34
12	0	4	38
13	0	4	42
14	0	4	46
15	0	4	50
16	0	4	54
17	0	4	58
18	0	4	62
19	0	4	66
20	0	4	70
21	-1	3	73
22	-1	2	75
23	-1	1	76
24	-1	0	76
25	0	0	76
26	0	0	76
27	0	0	76
28	0	0	76
29	0	0	76
30	0	0	76

INITIAL CONDITIONS



INPUT

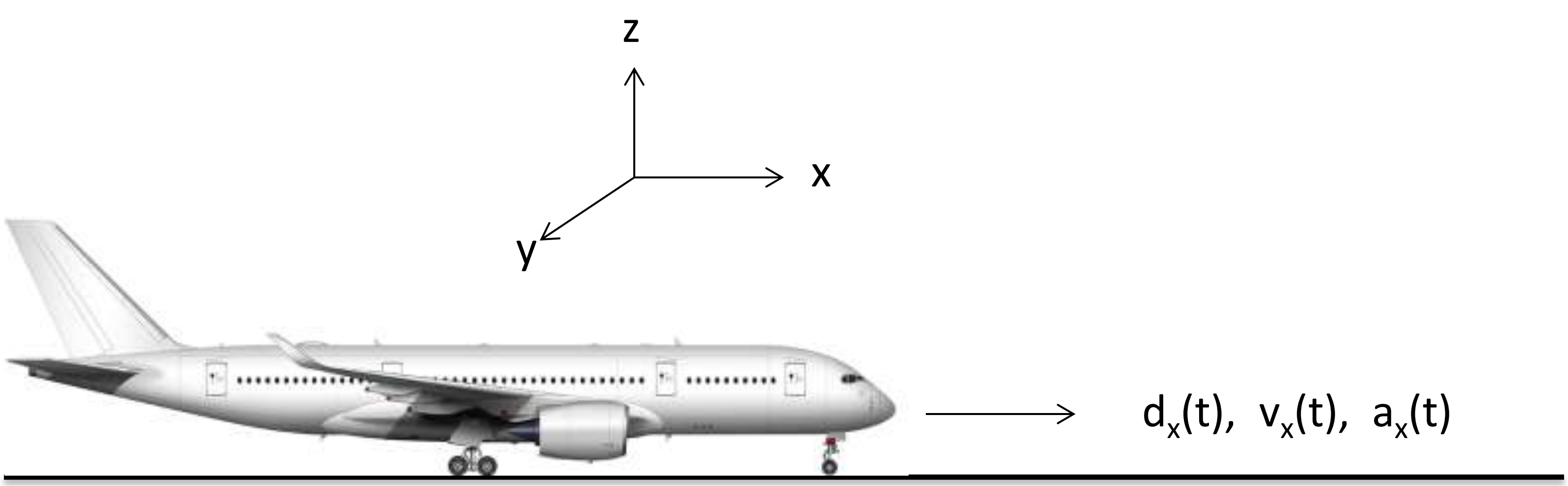
OUPUTS

Simulation

T (secs)	a(t) ft/sec/sec	v(t) ft/sec	d(t) ft
1	0	0	0
=A2+1	1	=C2+(B3*(A3-A2))	=D2+(C3*(A3-A2))
=A3+1	1	=C3+(B4*(A4-A3))	=D3+(C4*(A4-A3))
=A4+1	1	=C4+(B5*(A5-A4))	=D4+(C5*(A5-A4))
=A5+1	1	=C5+(B6*(A6-A5))	=D5+(C6*(A6-A5))
=A6+1	0	=C6+(B7*(A7-A6))	=D6+(C7*(A7-A6))
=A7+1	0	=C7+(B8*(A8-A7))	=D7+(C8*(A8-A7))
=A8+1	0	=C8+(B9*(A9-A8))	=D8+(C9*(A9-A8))
=A9+1	0	=C9+(B10*(A10-A9))	=D9+(C10*(A10-A9))
=A10+1	0	=C10+(B11*(A11-A10))	=D10+(C11*(A11-A10))
=A11+1	0	=C11+(B12*(A12-A11))	=D11+(C12*(A12-A11))
=A12+1	0	=C12+(B13*(A13-A12))	=D12+(C13*(A13-A12))
=A13+1	0	=C13+(B14*(A14-A13))	=D13+(C14*(A14-A13))
=A14+1	0	=C14+(B15*(A15-A14))	=D14+(C15*(A15-A14))
=A15+1	0	=C15+(B16*(A16-A15))	=D15+(C16*(A16-A15))
=A16+1	0	=C16+(B17*(A17-A16))	=D16+(C17*(A17-A16))
=A17+1	0	=C17+(B18*(A18-A17))	=D17+(C18*(A18-A17))
=A18+1	0	=C18+(B19*(A19-A18))	=D18+(C19*(A19-A18))
=A19+1	0	=C19+(B20*(A20-A19))	=D19+(C20*(A20-A19))
=A20+1	0	=C20+(B21*(A21-A20))	=D20+(C21*(A21-A20))
=A21+1	-1	=C21+(B22*(A22-A21))	=D21+(C22*(A22-A21))
=A22+1	-1	=C22+(B23*(A23-A22))	=D22+(C23*(A23-A22))
=A23+1	-1	=C23+(B24*(A24-A23))	=D23+(C24*(A24-A23))
=A24+1	-1	=C24+(B25*(A25-A24))	=D24+(C25*(A25-A24))
=A25+1	0	=C25+(B26*(A26-A25))	=D25+(C26*(A26-A25))
=A26+1	0	=C26+(B27*(A27-A26))	=D26+(C27*(A27-A26))
=A27+1	0	=C27+(B28*(A28-A27))	=D27+(C28*(A28-A27))
=A28+1	0	=C28+(B29*(A29-A28))	=D28+(C29*(A29-A28))
=A29+1	0	=C29+(B30*(A30-A29))	=D29+(C30*(A30-A29))
=A30+1	0	=C30+(B31*(A31-A30))	=D30+(C31*(A31-A30))

How to Compute <u>a(t)</u> for Aircraft on Airport Surface ?

- $\sum F(t) = mass * a(t)$



d_x = Distance covered in x direction
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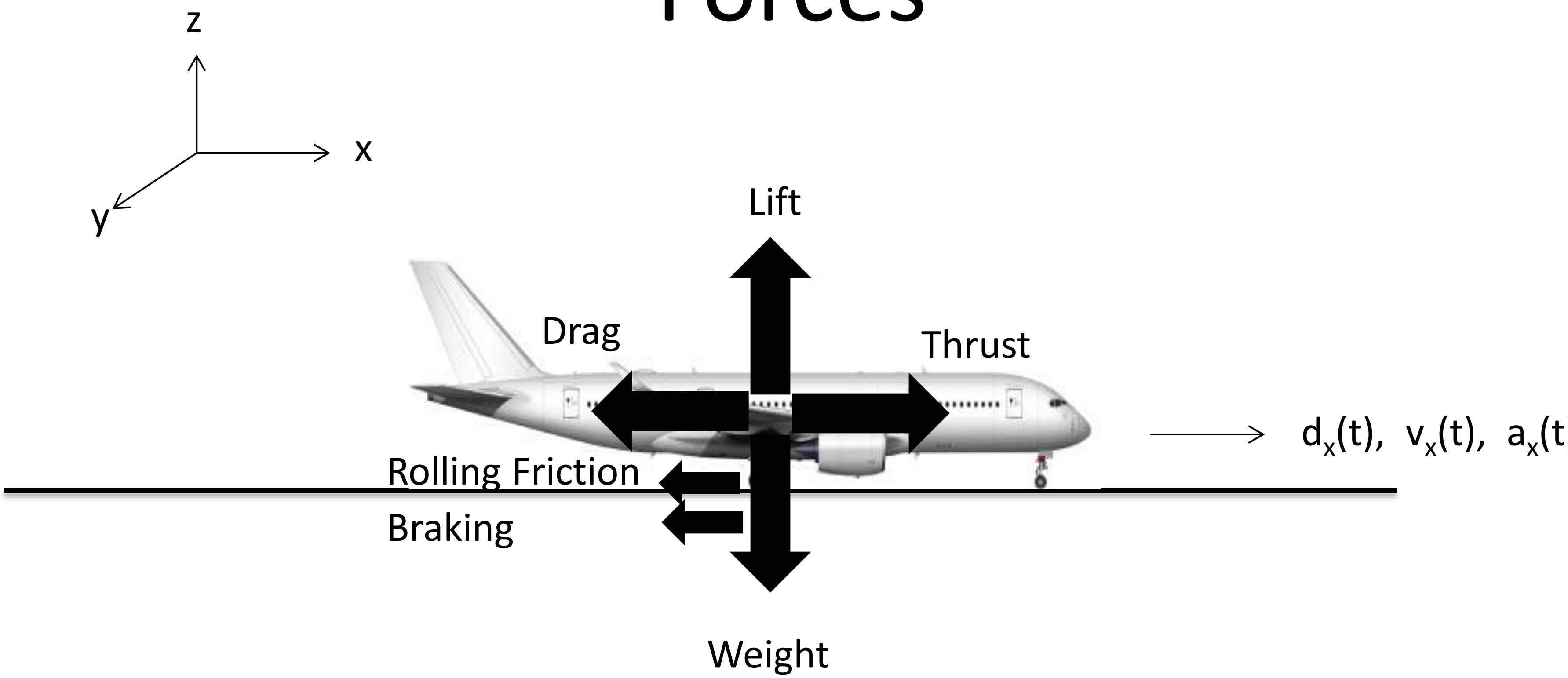
Forces



Lift – L (lbs) = ½ ρ S V^2 C_L – increases as speed increases above a threshold
Drag – D (lbs) = ½ ρ S V^2 C_D – increases as speed increases
Thrust – T (lbs) from engines
Weight – W (lbs) = mass * gravitational constant (32.2 ft/sec^2
Rolling Friction – μ (lbs) a function (W-L) – reduced as Lift is generated
Braking – B (lbs) – only in effect when brakes are applied

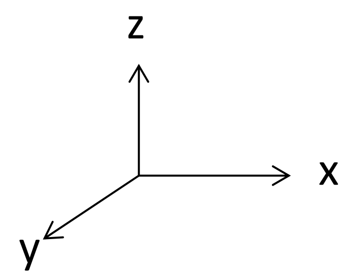
Sum of the Forces = ma(t)

$$\sum F(t) = \text{mass} * a(t)$$

Assumes Level surface

$$T - D - \mu - B = m * a(t)$$

$$(T - D - \mu - B)/m = a(t)$$



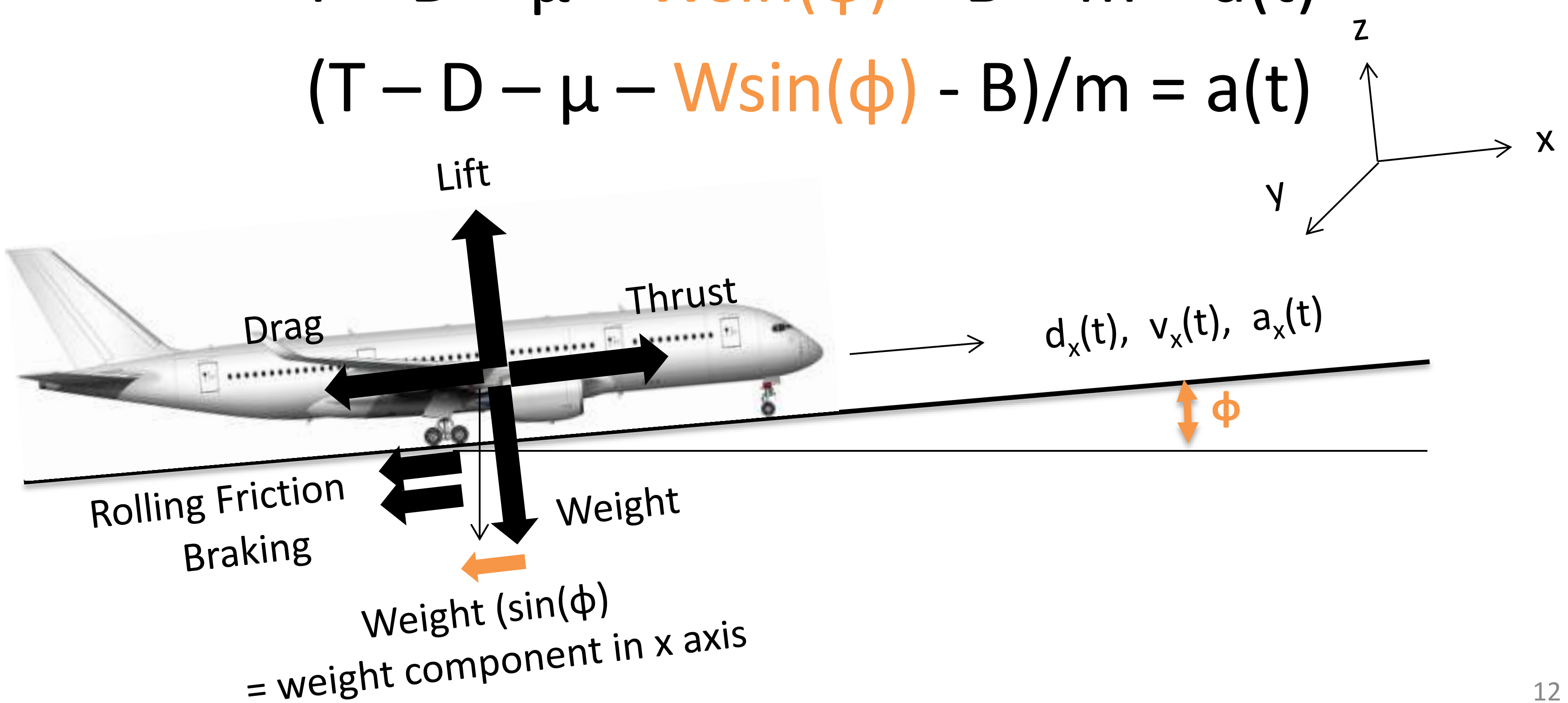
d_x = Distance covered in x direction
v_x = Velocity in x direction
a_x = Acceleration in x direction

Sum of the Forces for Incline= ma(t)

$$\sum F(t) = \text{mass} * a(t)$$

$$T - D - \mu - W\sin(\phi) - B = m * a(t)$$

$$(T - D - \mu - W\sin(\phi) - B)/m = a(t)$$



Test Yourself

1) How must Thrust does it take to cross the airport surface at a constant speed
2) How much additional Thrust does it take to accelerate (i.e. $a(t) > 0$)
3) Does it take more or less thrust to accelerate up an incline? How much more?

Test Yourself

1) How must Thrust does it take to cross the airport surface at a constant speed
2) How much additional Thrust does it take to accelerate (i.e. $a(t) > 0$)
3) Does it take more or less thrust to accelerate up an incline? How much more?

1) Thrust for constant speed (i.e. $a(t) = 0$): $T = D + \mu + W\sin(\phi)$
2) Thrust to accelerate: $T = \mathbf{ma(t)} + D + \mu + W\sin(\phi)$; difference is $ma(t)$ = inertia
3) Thrust for flat $\phi = 0$, $T = D + \mu$
Thrust for incline $\phi > 0$, $T = D + \mu + W\sin(\phi)$
Additional Thrust for incline = $W\sin(\phi)$ = overcome gravity

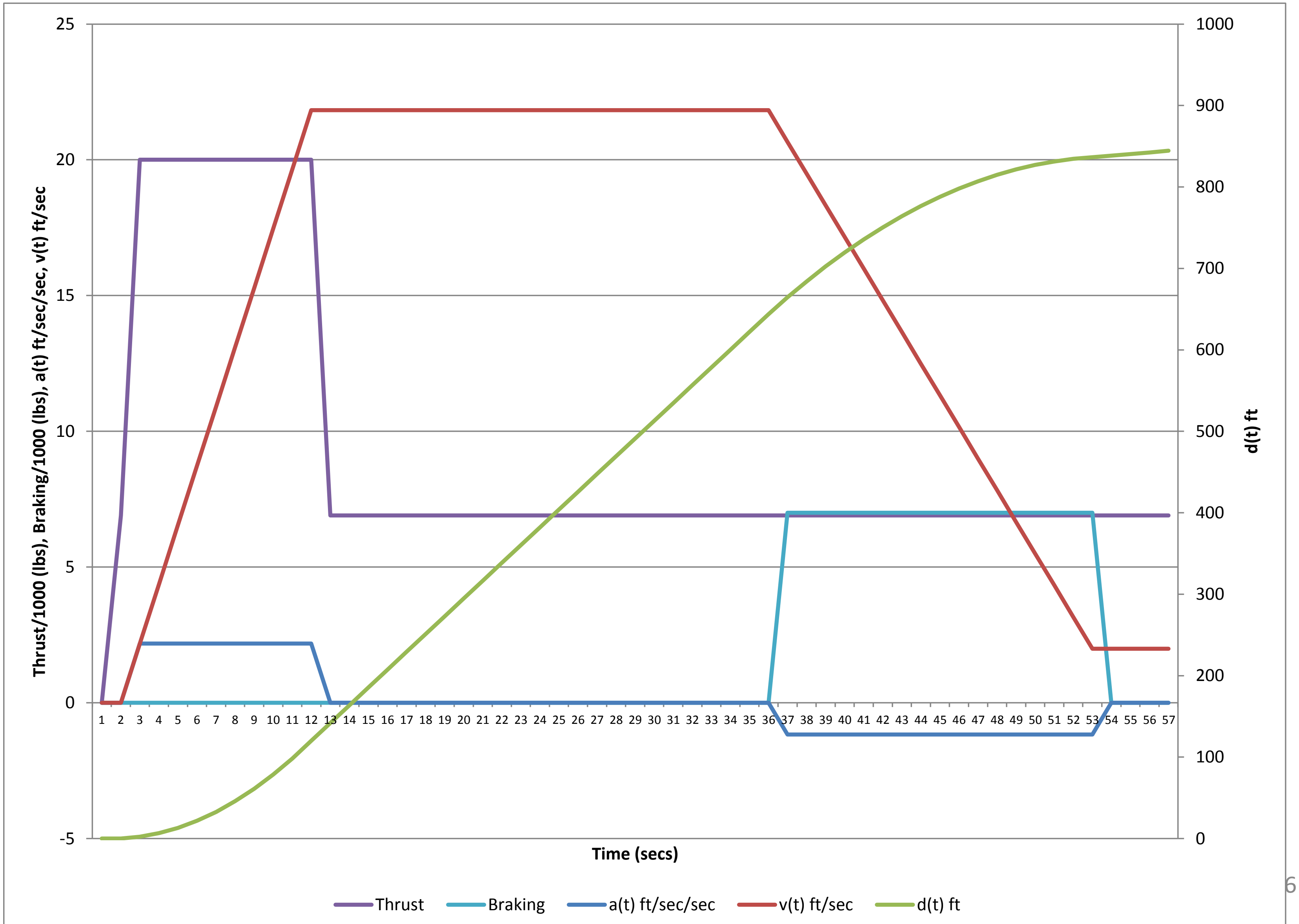
Simulation

Thrust (lbs)	Input
Drag (lbs)	6404
Lift (lbs)	0
Rolling Friction (lbs)	500
Mass (lbs)	6000
Gravitatonal Constant (ft/sec/sec)	32.2
Surface Incline	0

$a(t) = 1/m\,(T - D - \mu - W\sin(\phi))$

		INPUT	OUTPUTS					
T (secs)	**Brakes**	**Thrust**	**a(t) ft/sec/sec**	**a(t) g's**	**v(t) ft/sec**	**v(t) knots**	**d(t) ft**	**d(t) Nm**
0	0	0	0	0	0	0	0	0
1	0	6,904	0	0	0	0	0	0
2	0	20,000	2.182666667	0.067784679	2.1826667	1.293195	2.182667	0.000359
3	0	20,000	2.182666667	0.067784679	4.3653333	2.58639	6.548	0.001078
4	0	20,000	2.182666667	0.067784679	6.548	3.879585	13.096	0.002155
5	0	20,000	2.182666667	0.067784679	8.7306667	5.17278	21.82667	0.003592
6	0	20,000	2.182666667	0.067784679	10.913333	6.465975	32.74	0.005388
7	0	20,000	2.182666667	0.067784679	13.096	7.75917	45.836	0.007544
8	0	20,000	2.182666667	0.067784679	15.278667	9.052366	61.11467	0.010058
9	0	20,000	2.182666667	0.067784679	17.461333	10.34556	78.576	0.012932
10	0	20,000	2.182666667	0.067784679	19.644	11.63876	98.22	0.016165
11	0	20,000	2.182666667	0.067784679	21.826667	12.93195	120.0467	0.019757
12	0	6904	0	0	21.826667	12.93195	141.8733	0.023349
13	0	6904	0	0	21.826667	12.93195	163.7	0.026942
14	0	6904	0	0	21.826667	12.93195	185.5267	0.030534
15	0	6904	0	0	21.826667	12.93195	207.3533	0.034126
16	0	6904	0	0	21.826667	12.93195	229.18	0.037718
17	0	6904	0	0	21.826667	12.93195	251.0067	0.04131
18	0	6904	0	0	21.826667	12.93195	272.8333	0.044903
19	0	6904	0	0	21.826667	12.93195	294.66	0.048495
20	0	6904	0	0	21.826667	12.93195	316.4867	0.052087
21	0	6904	0	0	21.826667	12.93195	338.3133	0.055679
22	0	6904	0	0	21.826667	12.93195	360.14	0.059271
23	0	6904	0	0	21.826667	12.93195	381.9667	0.062864
24	0	6904	0	0	21.826667	12.93195	403.7933	0.066456
25	0	6904	0	0	21.826667	12.93195	425.62	0.070048
26	0	6904	0	0	21.826667	12.93195	447.4467	0.07364
27	0	6904	0	0	21.826667	12.93195	469.2733	0.077233
28	0	6904	0	0	21.826667	12.93195	491.1	0.080825
29	0	6904	0	0	21.826667	12.93195	512.9267	0.084417
30	0	6904	0	0	21.826667	12.93195	534.7533	0.088009
31	0	6904	0	0	21.826667	12.93195	556.58	0.091601
32	0	6904	0	0	21.826667	12.93195	578.4067	0.095194
33	0	6904	0	0	21.826667	12.93195	600.2333	0.098786
34	0	6904	0	0	21.826667	12.93195	622.06	0.102378
35	0	6904	0	0	21.826667	12.93195	643.8867	0.10597
36	7000	6904	-1.166666667	-0.036231884	20.66	12.24072	664.5467	0.10937
37	7000	6904	-1.166666667	-0.036231884	19.493333	11.54949	684.04	0.112579
38	7000	6904	-1.166666667	-0.036231884	18.326667	10.85826	702.3667	0.115595
39	7000	6904	-1.166666667	-0.036231884	17.16	10.16703	719.5267	0.118419
40	7000	6904	-1.166666667	-0.036231884	15.993333	9.475794	735.52	0.121051
41	7000	6904	-1.166666667	-0.036231884	14.826667	8.784563	750.3467	0.123491
42	7000	6904	-1.166666667	-0.036231884	13.66	8.093331	764.0067	0.125739
43	7000	6904	-1.166666667	-0.036231884	12.493333	7.4021	776.5	0.127796
44	7000	6904	-1.166666667	-0.036231884	11.326667	6.710869	787.8267	0.12966
45	7000	6904	-1.166666667	-0.036231884	10.16	6.019637	797.9867	0.131332
46	7000	6904	-1.166666667	-0.036231884	8.9933333	5.328406	806.98	0.132812
47	7000	6904	-1.166666667	-0.036231884	7.8266667	4.637175	814.8067	0.1341
48	7000	6904	-1.166666667	-0.036231884	6.66	3.945943	821.4667	0.135196
49	7000	6904	-1.166666667	-0.036231884	5.4933333	3.254712	826.96	0.1361
50	7000	6904	-1.166666667	-0.036231884	4.3266667	2.563481	831.2867	0.136812
51	7000	6904	-1.166666667	-0.036231884	3.16	1.872249	834.4467	0.137332
52	7000	6904	-1.166666667	-0.036231884	1.9933333	1.181018	836.44	0.13766
53	0	6904	0	0	1.9933333	1.181018	838.4333	0.137989
54	0	6904	0	0	1.9933333	1.181018	840.4267	0.138317
55	0	6904	0	0	1.9933333	1.181018	842.42	0.138645
56	0	6904	0	0	1.9933333	1.181018	844.4133	0.138973

Simulation


25
20
15
10
5
0
-5
Thrust/1000 (lbs), Braking/1000 (lbs), a(t) ft/sec/sec, v(t) ft/sec
1000
900
800
700
600
500
400
300
200
100
0
d(t) ft
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57
Time (secs)
Thrust
Braking
a(t) ft/sec/sec
v(t) ft/sec
d(t) ft

Simulation

	INPUT		OUTPUTS					
T (secs)	Brakes	Thrust	a(t) ft/sec/sec	a(t) g's	v(t) ft/sec	v(t) knots	d(t) ft	d(t) Nm
0	0	0	0	=K3/$B$6	0	=M3*0.592484	0	=O3*0.000164579
=H3+1	0	6904	=(1/$B$5)*(J4-$B$2-$B$4-(($B$5*$B$6)*SIN($B$7))-I4)	=K4/$B$6	=M3+(K4*(H4-H3))	=M4*0.592484	=O3+(M4*(H4-H3))	=O4*0.000164579
=H4+1	0	20000	=(1/$B$5)*(J5-$B$2-$B$4-(($B$5*$B$6)*SIN($B$7))-I5)	=K5/$B$6	=M4+(K5*(H5-H4))	=M5*0.592484	=O4+(M5*(H5-H4))	=O5*0.000164579
=H5+1	0	20000	=(1/$B$5)*(J6-$B$2-$B$4-(($B$5*$B$6)*SIN($B$7))-I6)	=K6/$B$6	=M5+(K6*(H6-H5))	=M6*0.592484	=O5+(M6*(H6-H5))	=O6*0.000164579
=H6+1	0	20000	=(1/$B$5)*(J7-$B$2-$B$4-(($B$5*$B$6)*SIN($B$7))-I7)	=K7/$B$6	=M6+(K7*(H7-H6))	=M7*0.592484	=O6+(M7*(H7-H6))	=O7*0.000164579
=H7+1	0	20000	=(1/$B$5)*(J8-$B$2-$B$4-(($B$5*$B$6)*SIN($B$7))-I8)	=K8/$B$6	=M7+(K8*(H8-H7))	=M8*0.592484	=O7+(M8*(H8-H7))	=O8*0.000164579
=H8+1	0	20000	=(1/$B$5)*(J9-$B$2-$B$4-(($B$5*$B$6)*SIN($B$7))-I9)	=K9/$B$6	=M8+(K9*(H9-H8))	=M9*0.592484	=O8+(M9*(H9-H8))	=O9*0.000164579
=H9+1	0	20000	=(1/$B$5)*(J10-$B$2-$B$4-(($B$5*$B$6)*SIN($B$7))-I10)	=K10/$B$6	=M9+(K10*(H10-H9))	=M10*0.592484	=O9+(M10*(H10-H9)	=O10*0.000164579
=H10+1	0	20000	=(1/$B$5)*(J11-$B$2-$B$4-(($B$5*$B$6)*SIN($B$7))-I11)	=K11/$B$6	=M10+(K11*(H11-H10))	=M11*0.592484	=O10+(M11*(H11-H1	=O11*0.000164579
=H11+1	0	20000	=(1/$B$5)*(J12-$B$2-$B$4-(($B$5*$B$6)*SIN($B$7))-I12)	=K12/$B$6	=M11+(K12*(H12-H11))	=M12*0.592484	=O11+(M12*(H12-H1	=O12*0.000164579
=H12+1	0	20000	=(1/$B$5)*(J13-$B$2-$B$4-(($B$5*$B$6)*SIN($B$7))-I13)	=K13/$B$6	=M12+(K13*(H13-H12))	=M13*0.592484	=O12+(M13*(H13-H1	=O13*0.000164579
=H13+1	0	20000	=(1/$B$5)*(J14-$B$2-$B$4-(($B$5*$B$6)*SIN($B$7))-I14)	=K14/$B$6	=M13+(K14*(H14-H13))	=M14*0.592484	=O13+(M14*(H14-H1	=O14*0.000164579

Test Yourself

1) How does one compute the amount of Thrust (input) for the simulation for a constant speed?

$T = D + \mu + W\sin(\phi)$

Additional Features to Sim

- Drag increases with Velocity
  - Drag – D (lbs) $= \frac{1}{2} \rho S V^2 C_D$
- Rolling Friction reduces with increased Lift
  - μ (lbs) a function (W-L)
  - Lift increases with velocity (above a threshold)
    - Lift – L (lbs) $= \frac{1}{2} \rho S V^2 C_L$
- Thrust → Fuel Burn → Emissions Inventory (Nox, Sox, HC, …)

Homework

- Build a simulation of a aircraft for movement on an airport surface
  - Simulate aircraft movement from stationary position to taxi (12 to 15 knots) to stationary over 0.25 nm
  - Show results in a Time plot
  - Use parameters →

Thrust (lbs)	Input
Braking Force (lbs)	Input
Drag (lbs)	6404
Lift (lbs)	0
Rolling Friction (lbs)	500
Mass (lbs)	6000
Gravitatonal Constant (ft/sec/sec)	32.2
Surface Incline	0