Aviation Safety: Homework & Solutions

- Please show all work
- Please refer to class handouts available on syllabus website

Question 1:

Consider 2 airlines, A and B. Each airline has 100,000 flights during the year. The total flight hours for Airline A are 150,000 hours. The total flight hours for Airline B are 200,000 hours.

Airline A has 3 fatal crashes with the following characteristics:
- Flight with 100 passengers, 2 passengers are killed
- Flight with 8 passengers, all 8 are killed
- Flight with 50 passengers, 5 passengers are killed

Airline B has 1 fatal crash with the following characteristics:
- Flight with 200 passengers, all 200 are killed

a) Compute the following metrics for both airlines.

Metric	Airline A	Airline B
a) Fatal accidents per flight hour	$3/150,000 = 2x10^{-5}$	$1/200,000 = 5x10^{-6}$
b) Fatal accidents per departure	$3/100,000 = 3x10^{-5}$	$1/100,000 = 1x10^{-5}$
c) Passenger fatalities per departure	$(2+8+5)/100,000 = 1.5x10^{-4}$	$(200)/100,000 = 2.0x10^{-3}$
d) Probability of being killed on a random flight (Barnett metric)	$(2/100)/100,000+(8/8)/100,000 + 5/50)/100,000 = 1.12x10^{-5}$	$(200/200)/100,000 = 1x10^{-5}$

b) Comment on the safety records of the 2 airlines. Would you not fly on either airline? Why?

Solution:

As a passenger, it would make sense to select metric (d) as this represents the passenger experience. Given that both Airline A and B have roughly the same values, there is no preference.

We need to be careful how we treat safety metrics as they are defined to support the position of the stakeholder.

Fatal accidents per flight hour-> Stakeholder = Aircraft Manufacturer
Fatal accidents per departure -> Stakeholder = Air Traffic Control
Passenger fatalities per departure - > Stakeholder = Airlines

Probability of being killed on a random flight by the airline- > Stakeholder = Passengers

Question 2:

Which of the following variables would be appropriate to model with a Poisson distribution? Explain what property(s) of Poisson distributions apply.

1. Number of fatal crashes in a year
2. Number of passenger fatalities in a year

Solution:

Number of fatal crashes in a year can be modeled as a Poisson distribution (crashes are rare, generally independent (aside from collisions), and there are a large number of flights).

Number of passenger fatalities in a year cannot be modeled as a Poisson distribution since passenger fatalities are not independent (knowing that one passenger was killed in a crash would mean that there are likely other passengers killed in the same crash).

Question 3:

Suppose there are 6,000,000 scheduled Part 121 airline flights per year. Suppose the probability of a fatal accident (per departure) is about 10^{-7}. What is the probability that there are 2 or more fatal accidents per year from this flight group?

Solution:

Let X_i = 1 if flight i is a fatal crash, 0 otherwise. $P(X_i = 1) = 10^{-7}$

$X = \sum_{i=1}^{6,000,000} X_i$ is approximately Poisson with mean 0.6

Then, $P(X \geq 2) = 1 - P(X < 2) = 1 - P(X = 1) - P(X = 2) = 1 - e^{-A} - Ae^{-A}$, where A = 0.6 (Answer: 0.12)

Question 4:

A startup airline has 2 fatal crashes in its first 1,000,000 flights. Thus, the estimated rate of fatal accidents per departure is 2×10^{-6}. Using a 95% Poisson confidence interval, determine whether or not you can conclude that the start-up airline is less safe than the industry average of 4×10^{-7} fatal accidents per departure.

Repeat the calculation using a 90% confidence interval.

Note 1: Use the CHINV function in Excel.
Note 2: α = .05 for 95% Confidence Interval. α = .1 for 90% Confidence Interval.

Solution:
The number of fatal crashes by the start-up airline is approximately a Poisson random variable.

- The observed value is 2.

For a 95% confidence interval, set α = .05.
- The lower and upper bounds for the true mean of this Poisson random variable are ... 0.24 and 7.22
- CHIINV(1 - 0.05/2, 2*2) / 2
- CHIINV(0.05/2, 2*2) / 2

The lowest safety rating we could assign to the airline is 0.24 / 1,000,000 = 2.4×10^{-7} which is below the industry average of 4×10^{-7}.
At 95% confidence you ... cannot conclude the airline is less safe than the industry average.

For a 90% confidence interval the lower bound (based on an observed value of 2) is (setting α = .1)

- CHIINV(1 - 0.1/2, 2*2) / 2 = 0.355.
- CHIINV(0.1/2, 2*2) / 2 = 6.3

The lowest safety rating assigned to the airline is 0.355 / 1,000,000 = 3.55×10^{-7} which is still below the industry average (but getting close).
At 90% confidence, you still cannot conclude the airline is less safe.

NOTES:

Probability of Accidents and the Poisson Distribution

The **Poisson distribution** is a discrete probability distribution. It expresses the probability of a number of events occurring in a fixed period of time if these events occur with a known average rate, and are **independent** of the time since the last event.

Examples of probability of a number of events:

a) The number of soldiers killed by horse-kicks each year in each corps in the Prussian cavalry
b) the number of V2 rocket attacks per area in England
c) the number of light bulbs that burn out in a certain amount of time.

The random variables *N* that count, a number of discrete occurrences that take place during a time-interval of given length. The probability that there are exactly *k* occurrences (*k* being a non-negative integer, *k* = 0, 1, 2, ...) is

$$f(k;\lambda) = \frac{e^{-\lambda}\lambda^k}{k!},$$

where

- *e* is the base of the natural logarithm (*e* = 2.71828...),
- *k* is the number of occurrences in a given time period
- *k*! is the factorial of *k*,
- λ is a positive real number, equal to the expected number of occurrences that occur during the given interval. For instance, if the events occur on average every 4 minutes, and you are interested in the number of events occurring in a 10 minute interval, you would use as model a Poisson distribution with λ=10/4=2.5.

The Poisson distribution can be derived as a limiting case of the binomial distribution.

$P(X=m) = (1-\lambda/n)^n = e^{-\lambda}$

Example application of Poisson Distribution:

Problem: An insurance company has five hundred automobile insurance policies. Assume that in a given year, the number of fatal automobile accidents has a binomial distribution. On average, there is one policy out of the five hundred that will be involved in a fatal crash. What is the probability that there will be no fatal accidents (out of five hundred policies) in any particular year?

Solution. Let X be the number of fatal accidents in a year from a population of 500 auto insurance policies.

p = average number of fatal accidents in year out of 500 policies = 1/500
n = 500 policies
λ = expected number of occurrences during a given period= 500 (1/500) = 1
$P(X=0) = e^{-1} = 0.368$
Using binomial distribution $P(X=m) = (\lambda^m/m!)\, e^{-\lambda}$ => $P(X=0) = (1-1/500)^{500} = 0.367$