Airline Scheduling Optimization
(Chapter 7 – I)

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Agenda

- Airline Scheduling
  - Factors affecting decision
  - Complexity and challenges
- Airline Schedule Planning Overview
  - Fleet Assignment Problem
    - Greedy Solution/Shortcomings/Need for Time-Space network
    - Fleet Assignment Mode
      - Basic FAM
      - Shortcomings of BasicFAM, Spill cost/recapture..
      - Extended FAM
      - IFAM (Itinerary Based )
  - Schedule Design Optimization
  - Crew and Maintenance Optimization Preview..
Objective of this Class
Objective of this Class

Assign Fleet Types to Each Leg using Optimization to Maximize Profit.

Output of “Schedule Design”
Factors affecting Airline Scheduling Decision (MACRO level)

- Market Demand (all PAX not same),
- Fleet composition,
- Location of crews,
- Maintenance bases,
  - $7.5 million last March against SWA. 46 B737 jets on 59,791 flights in 2006 and 2007 without mandatory fuselage inspections for fatigue cracking. Six planes had cracks, the FAA says. After SWA became aware it hadn't made the inspections, the airline continued to operate the 46 planes on an additional 1,451 flights.
- Gate restrictions,
- Landing slot restrictions (eg: NY airports),
- For International flights: bilateral agreements
Complexity of the Problem is affected by...

- **Airports are not similar**
  - Arr/Dep restrictions, Gates (type/personnel), Equipments..

- **Fleet composition**
  - Different operating characteristics, costs, maintenance and crew requirements, seating capacity …

- **Crews**
  - Crews capable of operating only certain aircraft types, Limitations of when/how they can work…

- **Different O-D markets**
  - Different demand volume, profitability/customer demographics..
Airline Schedule Planning challenges..

• STOCHASTIC problem,
  – Uncertainty in PAX demand, Pricing of tickets, Fuel, Crew availability, Weather …

• SIZE of problem
  – Break into sub problems and proceed..
Airline Schedule Planning

Each problem solved in order, with output of previous subproblem used as input for next subproblem.

Schedule Design

Fleet Assignment

Aircraft (Maintenance) Routing

Crew Scheduling

Select optimal set of *flight legs* in a schedule
(Flight legs to operate: Origin, Sch Dep Time, Approx Arrival Time, Frequency)

Assign aircraft types to flight legs such that *contribution* is maximized

Contribution = Revenue - Costs

Assign crew (pilots and/or flight attendants) to flight legs
The Fleet Assignment Problem

• Outline
  – Problem Definition and Objective
  – Fleet Assignment Network Representation
  – Fleet Assignment Model
Problem Definition

Given:

– Flight Schedule
  • Each flight covered exactly once by one fleet type
– Number of Aircraft by Equipment Type
  • Can’t assign more aircraft than are available, for each type
– Turn Times by Fleet Type at each Station
– Other Restrictions: Maintenance, Gate, Noise, Runway, etc. (Not addressed in formulation)
– Operating Costs, Spill and Recapture Costs, Total Potential Revenue of Flights, by Fleet Type
Problem Objective

Find:

- Cost minimizing (or profit maximizing) assignment of aircraft fleets to scheduled flights such that maintenance requirements are satisfied, conservation of flow (balance) of aircraft is achieved, and the number of aircraft used does not exceed inventory (in each fleet type)
Table 7.1

Example: flight schedule, fares and passenger demands

<table>
<thead>
<tr>
<th>Flight no.</th>
<th>From</th>
<th>To</th>
<th>Dep. time (EST)</th>
<th>Arr. time (EST)</th>
<th>Fare ($)</th>
<th>Demand (passengers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CL301</td>
<td>LGA</td>
<td>BOS</td>
<td>11:00</td>
<td>12:00</td>
<td>150</td>
<td>250</td>
</tr>
<tr>
<td>CL302</td>
<td>LGA</td>
<td>BOS</td>
<td>12:00</td>
<td>13:00</td>
<td>150</td>
<td>250</td>
</tr>
<tr>
<td>CL303</td>
<td>LGA</td>
<td>BOS</td>
<td>14:00</td>
<td>15:00</td>
<td>150</td>
<td>100</td>
</tr>
<tr>
<td>CL331</td>
<td>BOS</td>
<td>LGA</td>
<td>08:00</td>
<td>09:00</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>CL332</td>
<td>BOS</td>
<td>LGA</td>
<td>11:30</td>
<td>12:30</td>
<td>150</td>
<td>300</td>
</tr>
<tr>
<td>CL333</td>
<td>BOS</td>
<td>LGA</td>
<td>14:00</td>
<td>15:00</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>CL501</td>
<td>LGA</td>
<td>ORD</td>
<td>12:00</td>
<td>15:00</td>
<td>400</td>
<td>150</td>
</tr>
<tr>
<td>CL502</td>
<td>LGA</td>
<td>ORD</td>
<td>13:00</td>
<td>16:00</td>
<td>400</td>
<td>200</td>
</tr>
<tr>
<td>CL551</td>
<td>ORD</td>
<td>LGA</td>
<td>08:00</td>
<td>11:00</td>
<td>400</td>
<td>200</td>
</tr>
<tr>
<td>CL552</td>
<td>ORD</td>
<td>LGA</td>
<td>09:30</td>
<td>12:30</td>
<td>400</td>
<td>150</td>
</tr>
</tbody>
</table>

Output of “Schedule Design”

“Market”
Figure 7.1 and Table 7.2

Table 7.2  Fleet information

<table>
<thead>
<tr>
<th>Fleet type</th>
<th>No. of aircraft owned</th>
<th>Capacity (seats)</th>
<th>Per flight operating cost ($000)</th>
<th>LGA–BOS</th>
<th>LGA–ORD</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC9</td>
<td>1</td>
<td>120</td>
<td></td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>B737</td>
<td>2</td>
<td>150</td>
<td></td>
<td>12</td>
<td>17</td>
</tr>
<tr>
<td>A300</td>
<td>2</td>
<td>250</td>
<td></td>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>

\[ c_{l,f} = \text{fare}_l \times \min(D_l, \text{Cap}_f) - \text{OpCost}_{l,f} \]

with:

- \( c_{l,f} \): profitability of assigning fleet type \( f \) to flight leg \( l \);
- \( \text{fare}_l \): fare of flight leg \( l \);
- \( D_l \): demand of flight leg \( l \);
- \( \text{Cap}_f \): capacity of fleet type \( f \);
- \( \text{OpCost}_{l,f} \): operating cost of assigning fleet type \( f \) to flight leg \( l \).
Profit Calculation

Table 7.3  Profit ($000 per day)

<table>
<thead>
<tr>
<th>Flight no.</th>
<th>DC9</th>
<th>B737</th>
<th>A300</th>
</tr>
</thead>
<tbody>
<tr>
<td>CL301</td>
<td>8</td>
<td>10.5</td>
<td>22.5</td>
</tr>
<tr>
<td>CL302</td>
<td>8</td>
<td>10.5</td>
<td>22.5</td>
</tr>
<tr>
<td>CL303</td>
<td>5</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>CL331</td>
<td>8</td>
<td>10.5</td>
<td>7.5</td>
</tr>
<tr>
<td>CL332</td>
<td>8</td>
<td>10.5</td>
<td>22.5</td>
</tr>
<tr>
<td>CL333</td>
<td>8</td>
<td>10.5</td>
<td>7.5</td>
</tr>
<tr>
<td>CL501</td>
<td>33</td>
<td>43</td>
<td>40</td>
</tr>
<tr>
<td>CL502</td>
<td>33</td>
<td>43</td>
<td>60</td>
</tr>
<tr>
<td>CL551</td>
<td>33</td>
<td>43</td>
<td>60</td>
</tr>
<tr>
<td>CL552</td>
<td>33</td>
<td>43</td>
<td>40</td>
</tr>
</tbody>
</table>

LGA – BOS
Fare: 150
Demand: 250
Capacity(B737): 150
Operating Cost of B737 on LGA-BOS route: 12K

150*min(250,150) – 12k
= 10.5k

Greedy Approach
Greedy Solution and Shortcoming

- Static Network Representation is INSUFFICIENT to capture the ‘temporal nature’.
  – Solution is a Time-Space Network.
Figure 7.2

Figure 7.2 Time-space network
A300’s end up at different locations. Profit: 280,500
Figure 7.4

A300's end up at same location. Profit: 255,000
Figure 7.5 Fleet-specific time–space network with count time and wraparound ground arcs
Basic FAM

Minimize $\sum_{i \in F} \sum_{k \in K} c_i f_i^k$

subject to:

Serve All flight legs with exactly 1 fleet type

$$\sum_{k \in K} f_i^k = 1, \quad \forall i \in F \quad (7.1)$$

Balance at each Airport

$$y_{n^+}^k + \sum_{i \in O(k,n)} f_i^k - y_{n^-}^k - \sum_{i \in I(k,n)} f_i^k = 0, \quad \forall n \in N^k, \forall k \in K \quad (7.2)$$

Don’t exceed availability for each fleet type

$$\sum_{a \in CG(k)} y_a^k + \sum_{i \in CL(k)} f_i^k \leq M^k, \quad \forall k \in K \quad (7.3)$$

$$f_i^k \in \{0, 1\}, \quad \forall i \in F, \forall k \in K \quad (7.4)$$

$$y_a^k \geq 0, \quad \forall a \in G^k, \forall k \in K \quad (7.5)$$

Legend:

- $f_{i,k} = 1$, leg $i$ serviced by fleet $k$,
- $y_{n,a}^k =$ # of acft of type $k$ on ground arc $a$
- $M^k =$ # of aircrafts of fleet type $k$ available
- $N^k =$ Set of nodes for fleet $k$
- $G^k =$ set of ground arcs for fleet $k$
- $O(k,n)$ and $I(k,n) =$ set of flights originating and terminating at node $n$ in fleet $k$’s time-space network
- $CL(k)$ and $CG(k) =$ set of flight legs and ground arcs that cross the count time in fleet $k$’s network
Nodes = \{N1, N2, N3, N4, N5, N6, N7, N8\}  Arcs = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}

Ground Arcs = \{2, 4, 5, 8, 9\}  Flight Arcs = \{1, 3, 6, 7\}

i = \{L1, L2, L3, L4\}

k = \{1, 2\}  \{B757, DC90\}

M^1 = M^2 = 2

N^1 = N^2 = \{N1, N2, N3, N4, N5, N6, N7, N8\}

G^1 = G^2 = \{2, 4, 5, 8, 9\}

O(1, N1) = L1,  O(1, N3) = L2,  O(1, N5) = L3,  O(1, N6) = L4,  O(1, N2|N4|N7|N8) = null (Same for k = 2)

I(1, N2) = L1,  I(1, N4) = L2,  I(1, N8) = L3,  I(1, N7) = L4,  I(1, N1|N3|N5|N6) = null (Same for k = 2)

CG(1) = CG(2) = \{8, 9\}

CL(1) = CL(2) = \emptyset
Serve All Flight Legs (7.1)

\[ \sum_{k \in K} f^k_i = 1, \quad \forall i \in F \]

\[ f^1_{i=L1} + f^2_{i=L1} = 1 \]

\[ f^1_{i=L2} + f^2_{i=L2} = 1 \]

\[ f^1_{i=L2} + f^2_{i=L2} = 1 \]

\[ f^1_{i=L2} + f^2_{i=L2} = 1 \]
Balance Constraint (7.2)

\[ \sum_{i \in O(1, N1)} f^{k=1}_i - y^{k=1}_{a=N1^+} + \sum_{i \in I(1, N1)} f^{k=1}_i - y^{k=1}_{a=N1^-} = 0 \]

\[ \sum_{i \in O(1, N4)} f^{k=1}_i - y^{k=1}_{a=N4^+} + \sum_{i \in I(1, N4)} f^{k=1}_i - y^{k=1}_{a=N4^-} = 0 \]
Legend:

\( \text{CL}(k) \) and \( \text{CG}(k) \) = set of flight legs and Ground Arcs that cross the count time in fleet k’s network

\( \text{CG}(1) = \text{CG}(2) = \{8,9\} \)

\( \text{CL}(1) = \text{CL}(2) = \emptyset \)
Number of Variables

\[ i = \{L1, L2, L3, L4\} \]
\[ k = \{1, 2\} \]
\[ G1 = G2 = \{2, 4, 5, 8, 9\} \]

\[ f^k_i \in \{0, 1\}, \quad \forall i \in F, \forall k \in K \quad (7.4) \]
\[ y^k_a \geq 0, \quad \forall a \in G^k, \forall k \in K \quad (7.5) \]

\[ i(4) \times k(2) = 8 \quad ; \quad f \text{ Binary} \]
\[ a(5) \times k(2) = 10 \quad ; \quad y \text{ (automatically Integer because of balance and non-negativity constraints)} \]

\[ 10 + 8 = 18 \text{ variables} \]
FAM can be augmented with..

- Noise Restriction constraints
- Maintenance requirements
- Gate restrictions
- Crew considerations
Solution Time

• Table 7.4
Shortcoming of FAM

- Spill Cost and Recaptures ignored
- Consider only aggregate demand and average fares.
- Static demand is assumed (no seasonality etc considered)
Extending FAM: Introduction to Spilling
Example

\( (75, \$200) \rightarrow (150, \$225) \rightarrow (75, \$300) \) (Demand, Fare)

Max Possible Revenue

\[
= 75 \times 200 + 150 \times 225 + 75 \times 300 \\
= 71,250
\]

---

**Table 7.5 Demand data**

<table>
<thead>
<tr>
<th>Market</th>
<th>Itinerary (sequence of flights)</th>
<th>Number of passengers</th>
<th>Average fare</th>
</tr>
</thead>
<tbody>
<tr>
<td>X–Y</td>
<td>1</td>
<td>75</td>
<td>$200</td>
</tr>
<tr>
<td>Y–Z</td>
<td>2</td>
<td>150</td>
<td>$225</td>
</tr>
<tr>
<td>X–Z</td>
<td>1–2</td>
<td>75</td>
<td>$300</td>
</tr>
</tbody>
</table>

**Table 7.6 Seating capacity**

<table>
<thead>
<tr>
<th>Fleet type</th>
<th>Number of seats</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>100</td>
</tr>
<tr>
<td>B</td>
<td>200</td>
</tr>
</tbody>
</table>

**Table 7.7 Operating costs**

<table>
<thead>
<tr>
<th>Fleet type</th>
<th>Flight 1</th>
<th>Flight 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$10,000</td>
<td>$20,000</td>
</tr>
<tr>
<td>B</td>
<td>$20,000</td>
<td>$39,500</td>
</tr>
</tbody>
</table>

**Table 7. Possible fleeting configurations**

<table>
<thead>
<tr>
<th>Fleeting</th>
<th>Flight 1</th>
<th>Flight 2</th>
<th>Total operating cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>A</td>
<td>A</td>
<td>$30,000</td>
</tr>
<tr>
<td>II</td>
<td>A</td>
<td>B</td>
<td>$49,500</td>
</tr>
<tr>
<td>III</td>
<td>B</td>
<td>A</td>
<td>$40,000</td>
</tr>
<tr>
<td>IV</td>
<td>B</td>
<td>B</td>
<td>$59,500</td>
</tr>
</tbody>
</table>
Spilling

- FAM is leg-based
- Fares/PAX demand is itinerary (O-D pair) based
  Itinerary can be multiple legs. Leading to mismatch.

- Problem: Estimate “leg-bases Spill Costs”
  - Different methods:
    - Prorate total itinerary fare to flight legs s.t. their Sum equals total fare
      - Proration is typical done based on distance. Can also be done based on profitability, i.e. $/miles etc
      - Can also assign entire itinerary fare to each leg. Rationale: PAX will travel ALL or NO legs for any given itinerary

- Assumption: Airline has full discretion in determining which passenger it wishes to accommodate.
Revenue Maximizing Strategy for Spilling

• If Fleeting I is selected, i.e. Aircraft type A on both legs.
  - Available seats on each leg = 100
  - Demand in X-Y leg = 75 (from X-Y) + 75 (from X-Z) = 150
  - Demand in Y-Z leg = 150 (from Y-Z) + 75 (from X-Z) = 225
  - Need to spill 50 (150-100) and 125(225-100) PAX from leg 1 and 2 respectively
  - X-Z Fare (300) < X-Y Fare(200) + Y-Z Fare(225)
    - Spill 50 X-Z PAX first
    - X-Y leg is not beyond capacity now
    - As Fare Y-Z < Fare X-Z, spill (225-50-100) Y-Z PAX
Result Using Revenue Maximizing Strategy

Table 7.9 Minimum spill costs and resulting contributions for each fleeting combination

<table>
<thead>
<tr>
<th>Fleeting</th>
<th>Operating costs</th>
<th>Spilled passengers</th>
<th>Spill costs</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$30000</td>
<td>50 X–Z, 75 Y–Z</td>
<td>$31875</td>
<td>$9375</td>
</tr>
<tr>
<td>II</td>
<td>$49500</td>
<td>25 X–Z, 25 X–Y</td>
<td>$12500</td>
<td>$9250</td>
</tr>
<tr>
<td>III</td>
<td>$40000</td>
<td>125 Y–Z</td>
<td>$28125</td>
<td>$3125</td>
</tr>
<tr>
<td>IV</td>
<td>$59500</td>
<td>25 Y–Z</td>
<td>$5625</td>
<td>$6125</td>
</tr>
</tbody>
</table>

I: Contribution = Max Possible Revenue – (Spill + Operating Cost)
= 71250 – (50*300 + 75*225) + 31875
= 9375
Minimize Spill Cost for Each Flight Leg – Greedy Approach

Table 7.10  The contribution using a greedy algorithm

<table>
<thead>
<tr>
<th>Fleeting</th>
<th>Operating costs</th>
<th>Spilled passengers</th>
<th>Spill costs</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$30 000</td>
<td>50 X–Y, 125 Y–Z</td>
<td>$38 125</td>
<td>$3125</td>
</tr>
<tr>
<td>II</td>
<td>$49 500</td>
<td>50 X–Y, 25 Y–Z</td>
<td>$15 625</td>
<td>$6125</td>
</tr>
<tr>
<td>III</td>
<td>$40 000</td>
<td>125 Y–Z</td>
<td>$28 125</td>
<td>$3125</td>
</tr>
<tr>
<td>IV</td>
<td>$59 500</td>
<td>25 Y–Z</td>
<td>$5 625</td>
<td>$6125</td>
</tr>
</tbody>
</table>

I: Contribution = Max Possible Revenue – (Spill + Operating Cost)

= 71250 – ( 50*300 + 125*225) + 31875

= 3125
Need for Mathematical Models and Optimization Approaches..

- Enumeration of possible fleeting combinations for real scenarios is computationally expensive and sometimes even impossible.
  - AAL yielded annual improvement in revenue of .54 to .77%.
IFAM (Itinerary Based FAM) :
FAM with network effects
Expansion to basic FAM

- Include variables representing the mean number of PAX assigned to each itinerary in airline’s network
  - $t_{pr}$: Expected # of PAX desiring to travel on ‘p’ spilled to a different itinerary ‘r’

- Recapture rate:
  - $b_{pr}$: Estimated fraction of PAX spilled from ‘p’ and captured in itinerary ‘r’

- Therefore,
  - $b_p^p = 1$: All PAX desiring to travel on p accept that itinerary
  - $b_{pr} \cdot t_{pr} = \#$ of PAX traveling on ‘r’ that preferred ‘p’
Itinerary-Based FAM (IFAM)

Fleet Assignment

Consistent Spill + Recapture

Kniker (1998)
Problem Formulation

Let $P$ denote the set of itineraries, and let $\delta^r_i = 1$ if itinerary $r \in P$ contains flight leg $i \in F$, and $\delta^r_i = 0$ otherwise. These additional factors are added to the basic FAM to create the itinerary-based fleet assignment model, or IFAM, as follows:

Minimize $\sum_{i \in F} \sum_{k \in K} c^k_i f^k_i - \sum_{p \in P} \sum_{r \in P} \text{fare}_r b^r_p t^r_p$

subject to:

$\sum_{k \in K} f^k_i = 1, \quad \forall i \in F$

$y^k_n + \sum_{i \in O(k,n)} f^k_i - y^k_{n-} - \sum_{i \in I(k,n)} f^k_i = 0, \quad \forall n \in N^k, \forall k \in K$

$\sum_{a \in CG(k)} y^k_a + \sum_{i \in CL(k)} f^k_i \leq M^k, \quad \forall k \in K$

$\sum_{p \in P} \sum_{r \in P} \delta^r_i b^r_p t^r_p \leq \sum_{k \in K} CAP^k f_i, \quad \forall i \in F \quad (7.6)$

$\sum_{r \in P} t^r_p \leq D_p, \quad \forall p \in P \quad (7.7)$

$f^k_i \in \{0, 1\}, \quad \forall i \in F, \forall k \in K$

$y^k_a \geq 0, \quad \forall a \in G^k, \forall k \in K$

$t^r_p \geq 0, \quad \forall p \in P, \forall r \in P \quad (7.8)$
IFAM Augmentations

Minimize

\[ \sum_{i \in F} \sum_{k \in K} c_i^k f_i^k - \sum_{p \in P} \sum_{r \in P} \text{fare}_{r,p} b_{r,p}^r t_{p,r}^r \]

Operating Cost

Total Revenue

\[ \sum_{p \in P} \sum_{r \in P} \delta_i^r b_{p,r}^r t_{p,r}^r \leq \sum_{k \in K} \text{CAP}_k^k f_i^k, \quad \forall i \in F \]

Max Capacity of the fleet type servicing flight leg i

\[ \sum_{r \in P} t_{p,r}^r \leq D_p, \quad \forall p \in P \]

Unconstrained demand of P

Total # of PAX travelling on leg i

Total # of PAX travelling on or spilled from itinerary p

Variables
### Table 7.11  FAM vs. IFAM: problem size and solution times

Flight schedule: 2044 flight legs and 9 fleet types

<table>
<thead>
<tr>
<th>Problem size</th>
<th>FAM</th>
<th>IFAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of columns</td>
<td>18 487</td>
<td>77 284</td>
</tr>
<tr>
<td>No. of rows</td>
<td>7 827</td>
<td>10 905</td>
</tr>
<tr>
<td>No. of non-zero entries</td>
<td>50 034</td>
<td>128 864</td>
</tr>
<tr>
<td>Solution time (seconds)</td>
<td>974</td>
<td>&gt;100 000</td>
</tr>
</tbody>
</table>

IFAM vs FAM
Airline Schedule Planning

Select optimal set of flight legs in a schedule
(Flight legs to operate: Origin, Sch Dep Time, Approx Arrival Time, Frequency)

Assign aircraft types to flight legs such that contribution is maximized

Contribution = Revenue - Costs

Assign crew (pilots and/or flight attendants) to flight legs

Each problem solved in order, with output of previous subproblem used as input for next subproblem
Schedule Design Optimization

- Data might not be available for optimizing new schedule.
- Building new schedule from scratch may be computationally intractable.
- Dramatic changes to schedule not preferred as degree of consistency from one planning period to next, especially in business markets is highly valued.
Incremental Optimization

Also, not always possible to express ‘BEST’ schedule mathematically. (example..)

• Allow limited changes to a given/current schedule:
  – Airlines able to use historical booking data/traffic forecast
  – Required planning efforts and time manageable
  – Fixed investment at stations can be utilized efficiently (gate/aircraft lease agreements ..)
  – Consistency maintained for customers.

• Example: Retiming certain flight legs or replacing small set of unprofitable flight legs., redesigning airline hub connections...
Example: Hub Debanking

• Challenges posed:
  – Scheduling decision made for ALL flights legs, not just those at the hubs.
  – Fleeting decision renewed. Large/small example
  – Fleeting and Scheduling must be determined simultaneously. # of schedules is unlimited.
Optimizing Flight Retiming and Fleet Assignment Problem

• Special case of more generalized integrated schedule design and fleet assignment problem.
• Given: Set of flight legs to be operated
• Decision:
  – Flight retiming
  – Fleet Assignment
• Approach: In time-space network to include one flight arc copy for each possible departure time of each flight leg.
Formulation

\[ \text{Minimize} \quad \sum_{i \in F} \sum_{k \in K} \sum_{b \in B^i} c_{i,b}^k f_{i,b}^k \]

subject to:

\[ \sum_{k \in K} \sum_{b \in B^i} f_{i,b}^k = 1, \quad \forall i \in F \]

\[ y_{n+}^k + \sum_{(i,b) \in O(k,n)} f_{i,b}^k - y_{n-}^k - \sum_{(i,b) \in I(k,n)} f_{i,b}^k = 0, \quad \forall n \in N^k, \forall k \in K \]

\[ \sum_{a \in C \left( k \right)} y_a^k + \sum_{(i,b) \in C \left( l \right)} f_{i,b}^k \leq M^k, \quad \forall k \in K \]

\[ f_{i,b}^k \in \{0,1\}, \quad \forall i \in F, \forall b \in B^i, \forall k \in K \]

\[ y_a^k \geq 0, \quad \forall a \in G^k, \forall k \in K \]

\[ f_{i,b}^k = 1, \text{ if fleet type } k \text{ is assigned to operate leg } i \text{ and the departure time of leg } l \]

\[ \text{corresponds to the time of flight arc copy } 'b' \]
END Part I