Homework Solution:
Chapter 7

1.

Max Possible Revenue = 150*200 + 100*225 + 100*300
= 82500

Fleeting		Operating Cost	Spilled PAX			Spill Cost	Contribution
			X-Y	Y-Z	X-Z		
I	A-A	30000	50	0	50	25000	**27500**
II	A-B	49500	100	0	0	20000	13000
III	B-A	40000	0	0	50	15000	**27500**
IV	B-B	59500	50	0	0	10000	13000

Note: See the Solution spreadsheet for details.

In this particular example, both fleet I and III and optimal because they have maximum contribution of $27500 each.

Logic:
In this scenario, fare of X-Z(300) is < fare of X-Y(200) + fare of Y-Z(225). So, X-Z PAX are preferred to be spilled over any one of X-Y or Y-Z.

<u>Fleeting I (A-A)</u>
Demand for leg 1 (X to Y) is: 150(X-Y) + 100(X-Z) = 250
Demand for leg 2 (Y to Z) is: 100(Y-Z) + 100(X-Z) = 200

Demand for leg 1 (X to Y) is: 150
Capacity for leg 2(Y to Z) is: 150

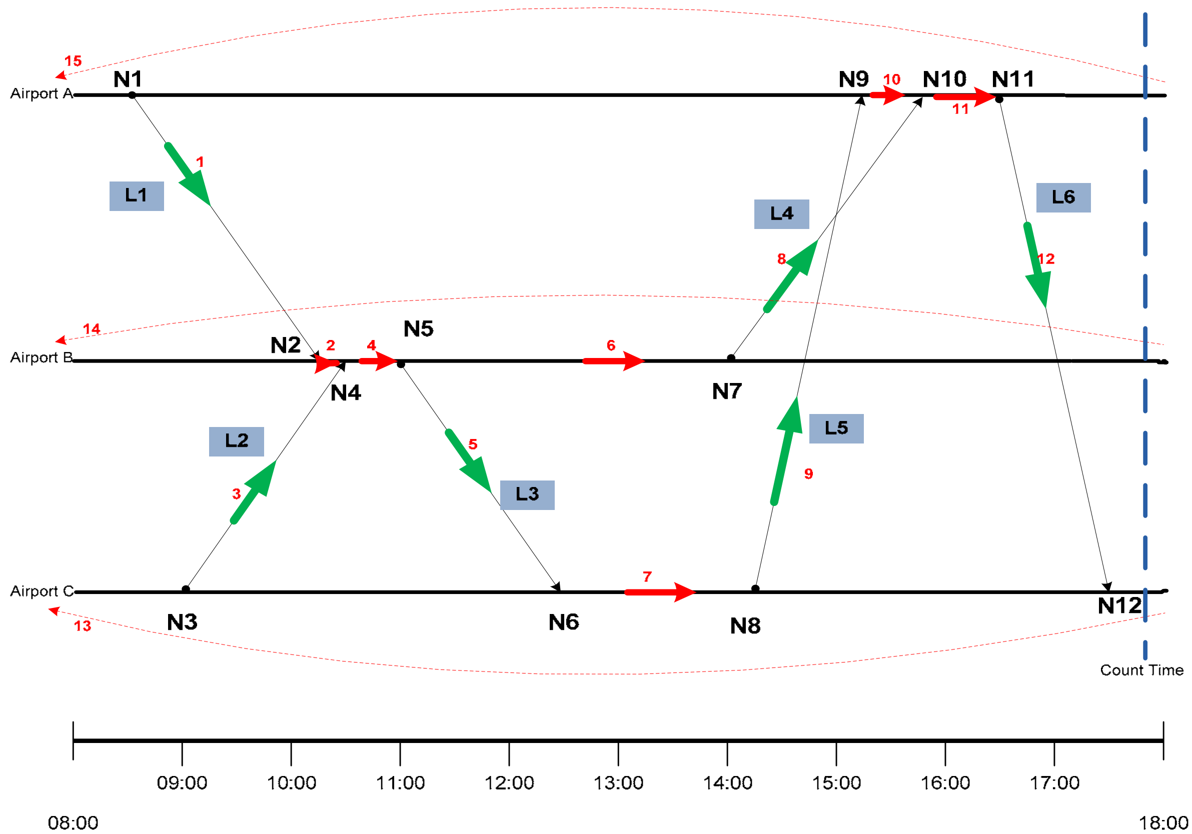
First we will spill PAX from the flight leg with lower demand, i.e. leg 2 (200<250). Number of PAX to be spilled is 50, (Demand – Capacity = 200-150). These 50 PAX will be X-Z PAX for reason mentioned above.

Now, the 'revised' demand for flight leg 1 is, 150(X-Y) + 50(X-Z). [This is because 50 X-Z pax that were spilled from leg 2 are now not going to travel at all]. We are still 50 over capacity. So 50 X-Y PAX will be spilled from this leg.

Spilled cost for this fleeting combination: 50(X-Y)*200 + 50(X-Z)*300 = 25000
Contribution = Max Possible Revenue – (Spilled Cost + Operating Cost)
= 82500 – (25000 + 30000)
= 27500

Similar logic can be used to obtain the contribution of other fleeting combinations.

2.



(a)
L1 = flight 301, L2 = flight 102, L3 = flight 101, L4 = flight302, L5 = flight 201, L6 = flight 202
I = {L1,L2,L3,L4,L5,L6}
k = {1,2}
G1 = G2 = {2,4,6,7,10,11,13,14,15}
i(6) * k(2) = 12 ; f binary
ground arcs (9)*k(2) = 18 ; y
12+18 = 30 variables

(b)
f^k_i is 1 if fleet k is assigned to flight leg i. It is 0 otherwise
y^k_a is number of aircraft of type k on the around arc a.

Nodes = {N1, N2, N3, N4, N5, N6, N7, N8, N9, N10, N11, N12}
Arcs = {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15}
Flight Arcs = {1,3,5,8,9,12}
Ground Arcs = {2,4,6,7,10,11,13,14,15}
i = {L1,L2,L3,L4,L5,L6}
k = {1,2} ---{B737,A320}
M1 = M2 = 2

N1 = N2 = {N1, N2, N3, N4, N5, N6, N7, N8, N9, N10, N11, N12}
G1 = G2 = {2,4,6,7,10,11,13,14,15}

O(1,N1) = L1, O(1,N3) = L2, O(1,N5) = L3, O(1,N7) = L4, O(1, N8) = L5,
O(1,N11) = L6, O(1, N2|N4|N6|N9|N10|N12) = Null

O(2,N1) = L1, O(2,N3) = L2, O(2,N5) = L3, O(2,N7) = L4, O(2, N8) = L5,
O(2,N11) = L6, O(2, N2|N4|N6|N9|N10|N12) = Null

I(1,N2) = L1, I(1,N4) = L2, I(1,N6) = L3, I(1,N10) = L4, I(1, N9) = L5,
I(1,N12) = L6, I(1, N1|N3|N5|N7|N8|N11) = Null

I(2,N2) = L1, I(2,N4) = L2, I(2,N6) = L3, I(1,N10) = L4, I(2, N9) = L5,
I(2,N12) = L6, I(2, N1|N3|N5|N7|N8|N11) = Null

CG(1) = CG(2) = {13,14,15}
CL(1) = CL(2) = null

C(1,1) = 13664.7, C(2,1) = 13571.52, C(3,1) = 13608, C(4,1) = 13624, C(5,1) = 7200,
C(6,1) = 6000
C(2,1) = 13500, C(2,2) = 13000, C(2,3) = 13000, C(2,4) = 13500, C(2,5) = 9000, C(2,6)
= 9000

Node	+	–
N1	0	15
N2	2	14
N3	0	13
N4	4	2
N5	6	4
N6	7	0
N7	14	6
N8	0	7
N9	10	0
N10	11	10
N11	15	11
N12	13	0

(c) MPL Code attached. Solution is:

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SOLUTION RESULT
  Optimal integer solution found
    MIN Z          =      67379.5200

DECISION VARIABLES

VARIABLE f[i,k] :
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i	k	Activity
1	1	0.0000
1	2	1.0000
2	1	1.0000
2	2	0.0000
3	1	1.0000
3	2	0.0000
4	1	0.0000
4	2	1.0000
5	1	1.0000
5	2	0.0000
6	1	1.0000
6	2	0.0000

(d)

If A320s can't fly to airport A. Then we will have to set all the f's for flight legs originating and terminating at airport A, i.e. L1,L4,L5 and L6 as 0. Mathematically this would be equivalent to adding constraints:

f(i=1,k=2) = 0
f(i=4,k=2) = 0
f(i=5,k=2) = 0
f(i=6,k=2) = 0

MPL CODE:

```
TITLE
	FAM_HWProblem;

INDEX
	i	:= 1..6;		!6 flight legs
	k	:= 1..2;	!2 fleet types
	N	:= 1..12;		!12 nodes
	a	:= 1..15;		!15 arcs/edges

	grA[a] := (2,4,6,7,10,11,13,14,15);!ground Arcs for A's network
	grB[a] := grA;

	flA[a]  := (1,3,5,8,9,12);! flight Arcs for A's(and B's) network
	flB[a]  := flA;

	N1[N] := N;    ! All the nodes in A's network
	N2[N] := N;

	CL	:= (0);  !CL and CG as defined on Pg 190 last paragraph
	CL1	:= CL;
	CL2	:= CL;

	CG[a]  := (13,14,15);
	CG1	:= CG;
	CG2	:= CG;
DATA
	nPlus[N]  := (0,2,0,4,6,7,14,0,10,11,15,13);
	nMinus[N] := (15,14,13,2,4,0,6,7,0,10,11,0);

	M[k]	:= (2,2); !total # of aircrafts of each type in inventory
	O[k,N]  := (1,0,2,0,3,0,4,5,0,0,6,0,
		1,0,2,0,3,0,4,5,0,0,6,0);

	I[k,N]  := (0,1,0,2,0,3,0,0,5,4,0,6,
	      0,1,0,2,0,3,0,0,5,4,0,6);

	c[i,k]	:= (13664.7,13500, 13571.52,13000, 13608,13000, 13624,13500,
7200,9000, 6000,9000);

BINARY VARIABLES
	f[i,k]; ! 1 if fight leg i is serviced by fleet type k

VARIABLES
	y[a,k]; ! # of aircraft of type k on ground arc a
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```
MODEL
	Min Z =  SUM(i,k:c*f);

SUBJECT TO

	cover[i]: SUM(k:f) = 1;

	bal[N,k]: y[a:=nPlus[N],k] - y[a:=nMinus[N],k] + SUM(i=O:f) - SUM(i=I:f) = 0;

	cp[k,CG,CL]: SUM(a=CG:y) + SUM(i=CL:f) <=M;
END
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