In *Proceedings of the International Conference on Research in Air Transportation*. Zilina, Slovaki, 45-51, 2004. Best Session Paper

Landing Safety Analysis of An Independent Arrival Runway

Yue Xie, George Mason University, Fairfax, Virginia, the United States

John Shortle, George Mason University, Fairfax, Virginia, the United States

Abstract

The growth of aviation traffic volume and the attempts to increase operational capacity by reducing aircraft spacing make aviation safety an urgent research issue in the aviation community. Simultaneous runway occupancy of landing aircraft is a precursor of a runway collision, and it is an effective of measure of runway landing safety. This paper presents an analytical method to evaluate the probability of a simultaneous runway occupancy using an $M/G/1$ queuing model. We conduct sensitivity analysis with the model to show the effects of reducing the mean and variance of runway occupancy time. The results show that landing safety can be significantly improved by reducing variance of occupancy time without changing the average length.

Introduction

Continuous growth in air traffic operations translates to increased stress on the National Airspace System, and especially on limited capacity areas of the air traffic infrastructure. The principal bottlenecks of the air transport system are the major airports both in the United States and Europe [1]. The terminal airspace and ground facilities of an airport, such as runways, taxiways, gates, etc., are relatively limited compared with the en-route airspace. The increased traffic volume will doubtlessly raise the delay both of arrivals and departures [2]. Intuitively, a larger number of airplanes in the terminal airspace will make collisions or other accidents more likely to happen.

Solutions to the flight delay problem have been widely discussed. One solution is to add runways to congesting airports. However, most congested airports are located in metropolitan areas, so that limited geographic space usually does not allow for more runways. Besides, the long period of infrastructure investment means that the benefit lies far in the future. Another method to increase capacity is to optimize the operating mechanism of the system. Improvement of air traffic control, both in technology and in operational procedures, has the potential to reduce delays by flying aircraft closer to each other. Reduction of wake vortex separation in the final approach phase is also a potential solution. In [3], the average 6% potential throughput increase achieved in the Dallas Aircraft VOrtex Spacing System (AVOSS) demonstration would result in as much as a 40% delay reduction at airports operating near capacity limits, such as Atlanta International Airport.

However, reduction of separation must not deteriorate flight safety. The arrival phase is the most accident-prose phase during a flight. According to the safety statistics for world-wide commercial jets from 1959 through 2003 [4], the final approach and landing phase accounts for 51% of accidents, while it only accounts for 4% of the flight time exposure.

Flying aircraft closer may increase the risk of a wake vortex encounter, air collision, and runway collision. Safety analysis related to vortex encounters can be found in [5]; [6]; [7].

An effective method to assess risk is to identify incidents that are direct precursors to accidents. For example, a simultaneous runway occupancy is a precursor to a runway collision. In this paper, we define a simultaneous runway occupancy to occur when an aircraft lands on a runway before the previous aircraft exits the same runway. With the occurrences of other general incidents, such as equipment failures or human performance deviations, the precursor can lead to an accident. An advantage of using incidents to analyze safety is that they can be more easily validated by empirical data.

The Generalized Reich model developed by Blom and Bakker in [8] on the basis of the Reich model ([9]) provides a mathematical method to evaluate the risk of collision, which is usually too small to obtain statistically using empirical observation.

This paper focuses on the landing safety analysis for an independent arrival runway. It begins with an overview of existing research, and then gives a simplified analytical model to predict the probability of a simultaneous runway occupancy.

In *Proceedings of the International Conference on Research in Air Transportation*. Zilina, Slovaki, 45-51, 2004.
Best Session Paper

Overview of Landing Safety Analysis

The National Airspace System (NAS) is predominately a hub-and-spoke network, with about 60 hub airports and a maximum capacity of about 40 million operations per year. Many hub airports are operating close to or even over their safe capacity. As an example, Figure 1 shows the airport capacity benchmark of ATL published by the FAA in 2001. It indicates that in VMC, the airport frequently accepts more than 90 aircraft per hour for the two arrival runways, and in IMC, 80 aircraft per hour. In both situations, the average time separation is not more than 90 seconds, which is less than the wake vortex separation requirements for most of the flight mixes. Although the average separation is larger than the average runway occupancy time, the probability of simultaneous runway occupancy may be larger than the intuitive estimation considering the variances of both runway occupancy time and separation.

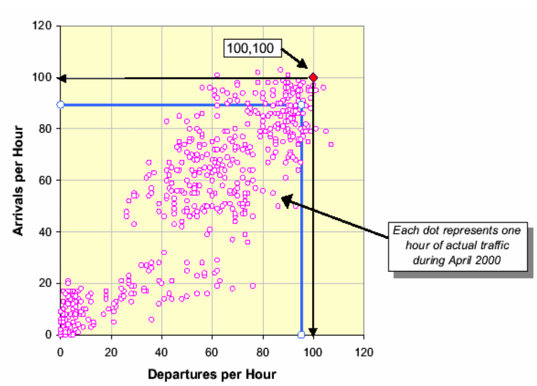

Figure 1. Arrival and Departure Rates of ATL

Lee ([10]) collected data on aircraft surface movements on the north side of ATL airport using the NASA Dynamic Runway Occupancy Measurement System (DROMS). The analysis shows that the runway occupancy time is strongly correlated with aircraft weight, landing speed, and it is further significantly influenced by runway conditions (dry or wet), wind directions (head wind or tail wind). The mean of runway occupancy time ranges from 35 seconds to 58 seconds, and the standard deviation from 4 to 12 seconds for different situations.

The variances of aircraft spacing and runway occupancy time make the occurrence of simultaneous runway occupancy possible. In an effort to look for the evidence of the tradeoff between safety and capacity, Hayine ([11]) collected data of landing time intervals (LTI) and runway occupancy times (ROT). The landing time interval is the time difference between two aircraft crossing the runway threshold. Haynie recorded 364 valid data points that have both LTI and ROT data. One case of simultaneous runway occupancy was observed. Figure 2 shows the frequency histograms of landing time interval and runway occupancy time. The overlapped area indicates a positive probability of simultaneous occupancy.

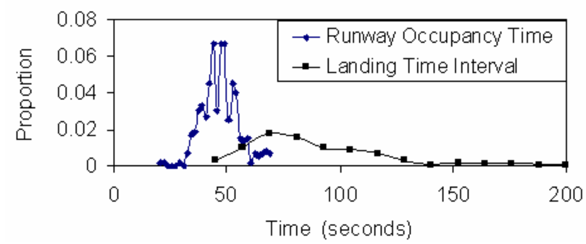

Figure 2. Haynie's Data of Arrivals in ATL

Cassell and Smith ([12]) have evaluated the probability of simultaneous runway occupancy though a simulation model. They first developed a fault tree for the runway occupancy scenario, identifying the causes related to hazards like simultaneous runway occupancy and runway collision. Then the landing time interval distribution was obtained by simulating various types of landing aircraft. The runway occupancy time distribution was obtained by mixing the distributions of aircraft leaving the runway from different exits. After that, they calculated the probability of simultaneous runway occupancy by convolution of the landing time interval distribution and the runway occupancy time distribution. The initial result was that the probability of simultaneous runway occupancy, given no intervention, was 0.034 per landing. The risk is considerably high, and the authors argued that if considering the go-around probability, the overall probability for heavy aircraft could be as low as 1.7×10^{-3} per landing.

Independent of the work of [12], Xie, Shortle and Donohue ([13]) estimated the probability of simultaneous runway occupancy for runway 26R at ATL airport based on Haynie's data ([11]). The result of the fast-time simulation model indicates an average probability of 0.0035 per landing for current traffic volume at ATL airport. The analysis used the post-go-around landing time interval data so the probability of missed approach was not considered.

The estimation of simultaneous runway occupancy assumes no intervention and that the distance of the follower from the leader is independent of the leader's runway occupancy time. The assumption seems to be valid because the data collected by Haynie show that the correlation coefficient between the landing time interval and the runway occupancy time is as small as 0.01.

The probability distribution functions of landing time interval and runway occupancy time are critical to calculate the probability of simultaneous runway occupancy. Runway occupancy time is commonly assumed to follow a Gaussian distribution, while it is more difficult to find an appropriate closed-form distribution to fit the landing time interval. Xie ([13]) used a Gaussian mixture distribution to approximate the distribution, and Levy ([14]) selected a Johnson SB curve. Although these curve fittings had satisfactory performance, they could not reveal the physical fundamentals of the process, and the approximation of the left tail (relatively shorter separation) was not good enough.

This paper is going to explain the distribution of landing time interval from the perspective of queuing theory because the distribution is actually the inter-departure distribution of the service phase in a queuing mode. Instead of using the method of curve fitting, the distribution will be obtained through the queuing model and described by a close-form function.

Methodology

Estimation of Landing Time Interval by Simulation

In this research, ATL is taken as an example for the case study since its runway layout is simple, its traffic volume is high, and we have a set of field observations of its runway operation. ATL has four inbound streams entering the TRACON area, and the streams merge into two final approach fleets landing on two arrival runways respectively, as shown in Figure 3 that is a snapshot taken from the software Flight Explorer.

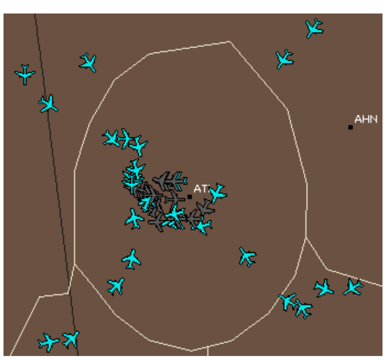

Figure 3. Snapshot of ATL TRACON. Courtesy of Flight Explorer

Since the two arrival runways in ATL are separated far enough, we can assume that the two runways are running independently. In addition, we assume each inbound aircraft chooses either runway to land randomly. Then the approach process can be modeled as two equivalent single-runway landing models, as shown in Figure 4.

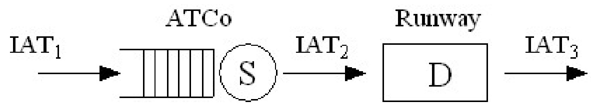

Figure 4: A Queuing Model of Arrival Process

IAT_1, IAT_2 and IAT_3 are variables for inter-arrival times between successive phases in the process. IAT_1 is the time interval between two aircraft entering the TRACON area. *S* is the separation time that an air traffic controller (ATCo) uses to separate two aircraft in a row passing the final approach fix. IAT_2 is the time interval of two aircraft touching down on the runway. *D* is the landing aircraft runway occupancy time. IAT_3 is the time interval between aircraft exiting the runway.

In this paper, we use a Poisson process to approximate the aircraft arrival process ([15]). The inter-arrival time in a Poisson process is exponentially distributed. Although the true arrival process may not be Poisson, we believe that the main results of this paper – namely, the estimation of IAT_2 and IAT_3 – are fairly robust to minor changes in the arrival process, holding the mean arrival rate fixed.

As for the service time in the queueing model, we need to understand the separation procedures of terminal approach. For simplicity, we suppose that in VFR, those aircraft that are close enough will attempt to maintain an optimum distance with each other under the permit of controllers. In IFR, controllers will instruct pilots to maintain at least required wake vortex separation. In each case, the separation is not deterministic because of the uncertainties in aircraft speed, perception deviation and weather disturbance. Normal distributions are often good at approximating errors, so we suppose the service time follows a Normal distribution. The difficulty lies in the estimation of mean and variance of the Normal distribution since we cannot directly observe the service time. A rough estimation can be obtained through the histogram of landing time intervals. From Figure 5, we can see that the distribution of landing time interval from Haynie's observation is heavily skewed to the right and the non-tail part displays a symmetric curve. The mean and variance of the service distribution can be estimated from the symmetric curve.

To verify the estimation, we put together Haynie's observation and simulation results of landing time interval from the queuing model in Figure 5, and they show a good match.

If aircraft runway occupancy time follows a Normal distribution with a mean of 48 seconds and a standard deviation of 8 seconds, the estimated probability of simultaneous runway occupancy with 95% confidence is located in the range (0.0026, 0.0033).

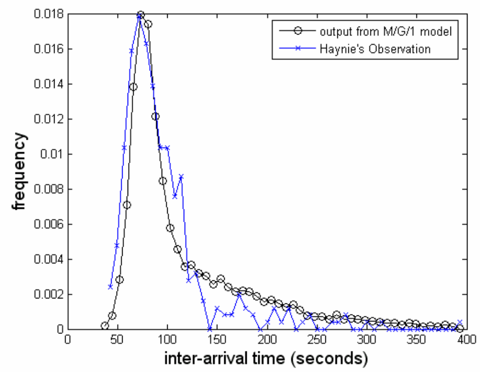

Figure 5. Landing Time Intervals from a M/G/1 Simulation Model and Haynies' Observation

Analytical Calculation of Landing Time Interval

Analytically estimating the probability of a simultaneous runway occupancy is challenging because we must determine the distribution function of inter-*departure* time of service (IAT_2 in Figure 4). If the server S is busy, the density function for the departure of the next aircraft from S is simply the same as the density function of service time. But if S is idle, an assumed observer standing at the runway threshold must first wait for an aircraft to enter the system and then for the aircraft to be processed by a time S. This is the convolution of the two probability density functions:

$$[p_1 \otimes p_2](t) \equiv \int_0^t p_2(h) p_1(t-h) dh \qquad (1)$$

where p_1 is the probability density function (pdf) of inter-arrival time, and p_2 is the pdf of service time.

The overall distribution is the weighted average of the two probabilities: one for the situation that S is idle, the other is for the situation that S is busy.

For an open *M/M/1* queue, which has exponentially distributed inter-arrival and service times, the pdf of inter-departure time is the same as that of inter-arrival time (assuming the utilization factor <1). In the case that >1, the pdf of inter-departure time has the same function with service time.

The queuing model in Figure 4 has a normal service time and Poisson arrival process, so it is a *M/G/1* model. To obtain an analytical form of the distribution function of inter-departure time needs a lot more work.

Acknowledging that many common pdf's can be approximated as a finite sum of terms of the form $x^k \exp(-\mu x)$ ([16]), we can view a system S that has m states as a network of exponential phases. We can use such a representation to approximate a normal distribution with arbitrary precision. A more detailed explanation of the following analysis can be found in [16]. The main result is Equation (10), below, which gives the analytical pdf for the inter-departure times from an *M*/*G*/1 queue.

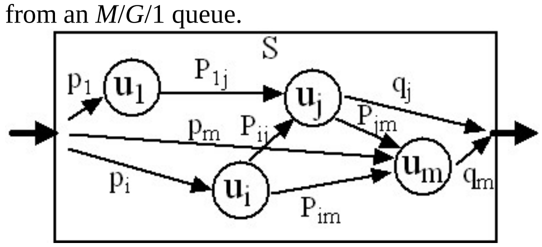

Figure 6. A Server with *m* Phases

Define P to be the transition matrix where P_{ij} is the probability that a customer will go from phase i to phase j when service at i is completed, q^T is an exit vector where q_i is the probability of leaving S when service at i is completed. It follows that

$$P\varepsilon^T + q^T = \varepsilon^T \qquad (2)$$

where ε^T is a column vector with all *1*'s. Define the completion rate matrix M as a diagonal matrix with elements M_{ii} = $_i$, where $_i$ is the rate of leaving state i. Then, we obtain the service *rate* matrix B as

$$B = M(I - P) \qquad (3)$$

The inverse $V = B^{-1}$ is called the service *time* matrix. Then, the mean service time of S is

$$\bar{x} = pV\varepsilon^T . \qquad (4)$$

Define the operator $\Psi[X] \equiv pX\varepsilon^T$, then

$$\bar{x} = \Psi[V]. \qquad (5)$$

The probability that a customer is still in S at time t, given that it was at phase i at time 0 is defined as $r_i(t)$, and its vector form is $r(t) = p \exp(-tB)$. Summing over all possible states, we get the probability that a customer is still in service at time t

$$R(t) = r(t)\varepsilon^T = p \cdot \exp(-tB)\varepsilon^T . \qquad (6)$$

It follows that the probability that a customer will leave S by time t, $S(t)$, is:

$$S(t) = 1 - R(t) = 1 - \Psi[\exp(-tB)]. \qquad (7)$$

Its derivative is the probability density function of service time, *s(t)*, which is

$$s(t) = \frac{dS(t)}{dt} = \Psi[B\exp(-tB)]. \qquad (8)$$

After obtaining the distribution of service time, we need to find out the distribution function of inter-departure time, IAT_2. In order to save time by using the process of obtaining the service time distribution, we can consider the arrival process as an exponential server with service rate , and integrate it with *S* to get a new server S', as shown in Figure 7.

The transition matrix P_d, corresponding to *P*, is easy to determine once we recognize that a customer enters the system S' through state *i* with probability p_i, and goes from state *i*>0 to *j* with probability P_{ij}. Therefore,

$$P_d = \begin{bmatrix} 0 & p \\ 0^T & P \end{bmatrix}, \text{ and } M_d = \begin{bmatrix} \lambda & 0 \\ 0^T & M \end{bmatrix}.$$

Then the process rate B_d, corresponding to service rate *B*, is

$$B_d = M_d(I_d - P_d) = \begin{bmatrix} \lambda & -\lambda p \\ 0^T & B \end{bmatrix}.$$

The process time matrix V_d is

$$V_d = B_d^{-1} = \begin{bmatrix} \frac{1}{\lambda} & pV \\ 0^T & V \end{bmatrix}.$$

As a result, the density function of inter-departure time *d(t)*, corresponding to *s(t)*, can be determined as

$$d(t) = p_d[B_d \exp(-xB_d)]\varepsilon^T = \Psi[B_d \exp(-xB_d)] \qquad (9)$$

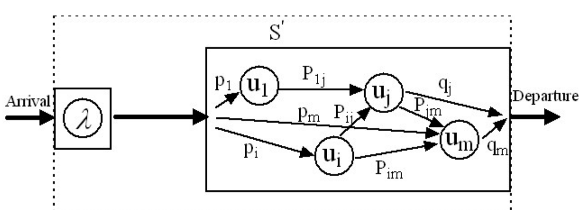

Figure 7. A Virtual Server with *m*+1 Phases

With some tedious linear algebra calculations, the final formula of *d(t)* is

$$d(t) = s(t) + (1-\rho) \times \left(\Psi[(I-\lambda V)^{-1}]\lambda e^{-x\lambda} - \Psi[(I-\lambda V)^{-1}B\exp(-xB)]\right) \qquad (10)$$

From Equation (10), we can see that when the utilization factor   goes to 1, the inter-departure time has the same distribution as the service time. When the traffic load is very light,   is close to 0, *(1- V)* is close to 1, and the inter-departure time distributes like the inter-arrival time with density function *e*$^{-\ x}$.

When the matrix exponential form of a service time distribution is known, *M* and *P* are determined, then *B* and *V* can be obtained according to Equation (3), and *d(t)* can be formed.

A normal-like distribution function of aircraft separation can be modeled as an Erlang distribution with parameters *r* and   with mean *r/* and variance *r/* An Erlang random variable is a sum of *r* exponential random variables, so when *r* is large, the distribution is approximately normal. In addition, the Erlang distribution is never negative (which is a problematic assumption for the normal distribution). Figure 8 gives some examples of the landing time interval distribution (IAT_2) calculated using Equation (10), using different Erlang functions that have different means and variances for time separation, given the same exponential arrival process

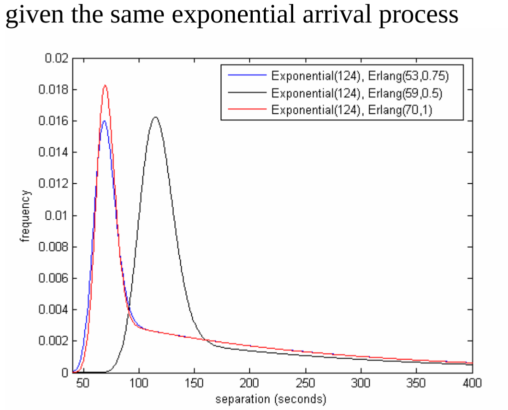

Figure 8. Landing Time Intervals with Different Configurations

Simultaneous Runway Occupancy Probability

As discussed above, the landing time interval distribution is a function of traffic volume and separation strategy. Given the distribution of aircraft landing runway occupancy time, the probability of simultaneous runway occupancy, can be calculated as a convolution of the two distributions, based on the assumption that they are independent.

$$\begin{aligned} &\text{Prob(SRO)} = \text{Prob(LTI<ROT)} \\ &= \int_{-\infty}^{0}\int_{-\infty}^{+\infty} f_{LTI}(x+z) f_{ROT}(x)\,dx\,dz \end{aligned} \qquad (11)$$

where $f_{LTI}(x)$ and $f_{ROT}(x)$ are density function of landing time interval and runway occupancy time respectively. The analytical value of $f_{LTI}(x)$ can be determined given traffic volume and separation strategy, which includes desired average separation and its deviation. We assume $f_{ROT}(x)$ follows a normal distribution, whose mean and variance depends on aircraft weight and speed, runway exit positions, and runway surface. To illustrate the application, we calculate the probability of simultaneous runway occupancy given that average inter-arrival time is 124 seconds ([15]), desired separation is 80 seconds, separation deviation is 11 seconds, average runway occupancy time is 48 seconds, and occupancy time deviation is 8 seconds ([11]). The probability is 0.00312, which is in the 90% confidence level estimation in ([13]), and is very close to the estimation in [15].

The probability of simultaneous runway occupancy is sensitive to changes of the arrival rate, desired separation, separation variance, mean of runway occupancy time, and variance of runway occupancy time. To illustrate the sensitivity to runway occupancy time, we calculate the probability of simultaneous runway occupancy with a variety of means and deviations of runway occupancy time, given a specified traffic volume and separation distribution, as shown in Figure 9 and 10.

Figure 9 gives contours of probability of simultaneous runway occupancy. For each combination of mean and variance of runway occupancy time, the corresponding safety value can be read from the contours. An overview of the relationship among safety (i.e. probability of simultaneous runway occupancy), average occupancy time, and its deviation is shown in Figure 10. We can observe an obvious nonlinear pattern.

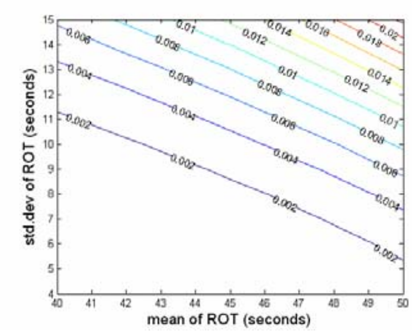

Figure 9. Contours of Prob(SRO) with Different ROT Properties

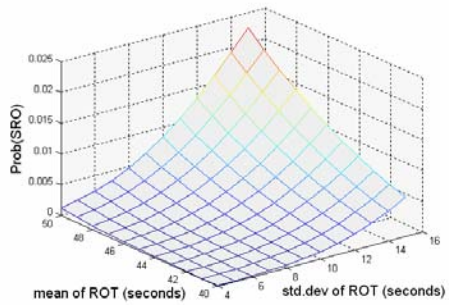

Figure 10. An Overview of Relationship Between Landing Safety and ROT

Conclusion

This paper presents a method of using an *M/G/1* queuing model to evaluate the probability of simultaneous runway occupancy. Traditional queuing theory focuses on analysis of the queue length and waiting time, and it has not been applied to aviation safety analysis.

Although simulation models can give good estimations of landing safety, a valid analytical model can help us to understand the underlying physical principles of the system. Furthermore, it is easier and faster to use an analytical model to conduct sensitivity analysis. In addition, the simple queueing model in this paper gives results which are in good agreement with field observations of landing time intervals (as shown in Figure 5).

This paper illustrates a non-linear relationship between safety and runway occupancy time, including its average value and variance. The analysis shows that a variance reduction of occupancy time can significantly decrease the probability of simultaneous runway occupancy.

Acknowledgments

This work has been partially supported by the WakeVAS program at NASA Langley Research Center, under Wayne Bryant, and the National Institute for Aerospace, contract NAS1-02117.

References

[1] Terrab, Mostafa, and Amedeo R. Odoni, 1993, *Strategic Flow Management For Air Traffic Control*. Operations Research, Vol. 41, No. 1, pp. 138-152.

[2] Peterson, Michael, Dimitris Bertsimas, and Amedeo Odoni, 1995, *Decomposition Algorithms for Analyzing Transient Phenomena in Multiclass*

Queuing Networks in Air Transportation. Operations Research, Vol.43, No.6.

[3] O'Connor, Cornelius, and David Rutishauser, 2001, *Enhanced Airport Capacity Through Safe, Dynamic Reductions In Aircraft Separation: NASA's Aircraft VOrtex Spacing System (AVOSS).* Journal of Air Traffic Control, Volume 43, No. 3, pp. 4-10.

[4] Boeing, 2001, *Air Traffic Management Revolutionary Concepts That Enable Air Traffic Growth While Cutting Delays.* The Boeing Company, Ref 291063.

[5] National Transportation Safety Board, 1994, *Safety Issues Related to Wake Vortex Encounters during Visual Approach to Landing.* Special Investigation Report NTSB/SIR-94/01, Washington, D.C.

[6] Veritas, Det Norske, 2002, *RDSM Safety Assessment – Part 1.* DFS, Germany.

[7] Kos, J., H. Blom, et al., 2001, *Probabilistic Wake Vortex Induced Accident Risk Assessment.* In: Air Transportation Systems Engineering, G.L. Donohue and A.G. Zellweger (Eds.), AIAA, 2001, pp. 513-531.

[8] Bakker, G. J. and H.A.P. Blom, *Air Traffic Collision Risk Modeling,* Proceedings of the 32nd IEEE Conference on Decision and Control, Vol.2, Institute of Electrical and Electronics Engineers, New York, 1993, pp. 1464 – 1469.

[9] Reich, P.G., 1966, *Analysis of Long-Range Air Traffic Systems, Separation Standards-I.* Journal of Navigation, Vol. 19.

[10] Lee, Derrick D., Alexander Smith, Rick Cassell, and Bassam Abdul-Baki, 1999, *NASA Low Visibility Landing and Surface Operations (LVLASO) Runway Occupancy Time (ROT) Analysis*, AIAA 18th Annual Digital Avionics Systems Conference.

[11] Haynie, R., 2002, *An Investigation Of Capacity And Safety In Near-Terminal Airspace For Guiding Information Technology Adoption.* doctoral dissertation, George Mason University, 2002.

[12] Cassell, Rick, Alex Smith, and Roger Shepherd, 1996, *A Risk Assessment Model for Free Flight - Terminal Area Reduced Separation*, the 15th AIAA/IEEE Digital Avionics Systems Conference in Atlanta, October, 1996.

[13] Xie, Yue, John Shortle, and George Donohue, 2003, *Runway Landing Safety Analysis: A case study of Atlanta Hartsfield Airport.* The 22nd AIAA/IEEE Digital Avionics Systems Conference in Indianapolis, October, 2003.

[14] Levy, B., J. Legge, and M. Romano, 2004, *Opportunities for Improvements in Simple Models for Estimating Runway Capacity*, to be presented at the 23rd Digital Aviation Systems Conference, Salt Lake City, Utah.

[15] Xie, Yue, John Shortle, and George Donohue, *Airport Terminal-Approach Safety and Capacity Analysis Using an Agent-Based Model*, to appear in Proceedings of the 2004 Winter Simulation Conference, 2004

[16] Lipsky, Lester, 1992, *Queuing Theory, a Linear Algebraic Approach*. Macmillan Publishing Company.

Biography

YUE XIE is currently a Ph.D candidate in Department of Systems Engineering and Operations Research, George Mason University. His research interests include modeling and simulation of aviation system and the application on aviation safety analysis. He received his M.S. in 2001 and B.S. in 1998 both from HuaZhong University of Science and Technology, China. His e-mail address is <yxie@gmu.edu>.

JOHN SHORTLE is currently an assistant professor of systems engineering at George Mason University. His research interests include simulation and queueing applications in telecommunications and air transportation. Previously, he worked at US WEST Advanced Technologies. He received a B.S. in mathematics from Harvey Mudd College in 1992 and a Ph.D. and M.S. in operations research at UC Berkeley in 1996. His e-mail address is <jshortle@gmu.edu>.