WHY EQUITY IS SO ELUSIVE:
DYNAMICAL PROPERTIES OF OVERSCHEDULED
NATIONAL AIRSPACE SYSTEM (NAS) RESOURCES

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Abstract: One of the major issues in the proposed concepts-of-operations for NEXTGEN is the equitable allocation of available National Airspace System (NAS) resources when demand is in excess of the capacity of the resource. NAS resources include airport and airspace slots allocated to flights from different airlines, carrying passengers and cargo from different geographic areas, affecting local air and water quality, and choosing different strategies for utilization of fossil fuels that affect global climate change.

Although federal regulations, Congressional policies, and modernization plans call for equitable allocation of publicly held resources, the mechanisms for equitable allocation, and the trade-offs that must be made between stakeholders, exhibit a high degree of social, political, and economic complexity. Further, the allocations that are routinely made in Traffic Flow Management (TFM) and Air Traffic Control (ATC) are subject to widespread perceptions that the allocations are not systemically equitable. The purpose of this paper is to establish the underlying properties of overscheduled resources and the associated limits on equitable allocation. These theoretical results should form the starting point for the development of feasible, robust, sustainable, NAS policies.

This paper describes the dynamic properties of a deterministic queueing model when demand for a resource is scheduled in excess of its capacity. The model demonstrates that (i) a natural asymmetry exists in the allocation of over-scheduled resources due to a “spill-over effect,” (ii) this asymmetry results in a low probability (< 10%) of an equitable allocation of resources amongst groups of flights (e.g. airlines), and (iii) increased competition reduces the feasibility of equitable allocations. The implications of these results are that the equitable allocation of a resource scheduled by independent groups (i.e. airlines) is very difficult to achieve. Expectations for equity in modernization plans should be tempered to meet this reality. These and other implications for planning, policy, and operations in a next-generation NAS are discussed.

INTRODUCTION

Agencies responsible for air transportation in Europe and the United States have announced long-term plans to significantly increase the capacity and productivity of the air transportation system through a process of modernization of the technologies and procedures used by Air Navigation Service Providers (ANSPs). In Europe the plans are
known as SESAR (SESAR Consortium, 2008). In the United States the plans are known as NEXTGEN (JPDO, 2007).

Both of these plans call for equitable access and use of public airspace and airport resources in several dimensions. For example, the plans for NextGen describe “equitable treatment of flight operators” (pg 2-16), “equitable access to NAS resources—both airports and airspace [for all users]” (pg 2-29), and “equitable management of system resources” (pg C-7). Whereas these ideas are central to democratic ideals of governing in the “best interests of all citizens” (RTCA, 2002), the reality of equitable allocation of scarce resources is elusive (Hoffman & Davison, 2003; Litman, 2006) and involves trade-offs between stakeholders (Manley & Sherry, 2008; Schaar, Sherry; 2009). Although there is no publicly available analysis, the allocation of flight delays during Traffic Flow Management (TFM) initiatives (i.e. such as Ground Delay Programs and Airspace Flow Programs) is the subject of much debate in the hallways and cafeterias of airline, business jet operators, and general aviation users, as well as, the ANSP providers. As one senior ANSP manager put it, “we feel like we have done a good job when everybody is equally unhappy.”

This paper describes an analysis of the properties of the dynamics of resource allocation when the demand for a scarce resource is scheduled in excess of its capacity. This situation occurs predominantly in the U.S. where regulations allow for airlines to schedule departures, arrivals and routes without consideration of other users of the required resources. The model identified the following major properties that result from the allocation of any scarce resources in the presence of over-utilization:

1. **Asymmetry in Allocation** – the natural dynamics of overscheduled resources is that flights later in the overscheduled time period will receive more delays than those in the front of the queue. This is a property of the spill-over effects of over-schedules resources.

2. **Collateral Damage** - Flights that are scheduled after the congested time period are also likely to receive (significant) delays due to spill-over effects.

3. **Probabilities of Achieving Proportional Equity Amongst Users is Very Low** - equity for groups of flights (e.g. airlines, geographic access, environmental parameters), measured as the allocation of scarce resources in proportion to the number of flights in each group, can be achieved only by rigid spacing of flights in the schedule. Ignoring the requirements for airline networking (e.g. arrival and departure banks), the combinatorial probabilities for equitable allocation amongst groups of users is less than 10% when all users have the same number of flights. When the distribution of flights amongst users is not proportional, the probability of equitable allocation of scarce resources is much less than 10%.

4. **Increasing Competition, Decreases Opportunities for Equity** – as the number of users increases (i.e. increased competition), the probability of scheduling with spacing to achieve equity decreases rapidly.
These results establish the boundaries of a “feasible-space” in which equitable allocation can occur. The implications of these results are that equitable allocation of a randomly scheduled resource is close to impossible and therefore need to be specified very carefully in modernization plans.

This paper is organized as follows. The properties of the dynamics of the queues when the demand for scarce resources in excess of the available capacity is described in Section 2. Section 3 provides a definition of proportional equity and describes the properties of equitable allocation. Section 4 uses these properties to derive the probability of an equitable allocation of resources amongst groups of users (e.g. airlines). The implications of these results are discussed in the Conclusions, Section 5.

2 DYNAMICAL PROPERTIES OF OVERSCHEDULED RESOURCES

A canonical representation of the scheduled over-utilization of a scarce resource is illustrated in Figure 1. In this scenario, flights are scheduled in 15 minutes periods. Starting at 15 minute period #6, flights are scheduled in excess of the available capacity for a period known as the congested period. Following the congested period, the low-demand provides a reservoir to absorb spill-over delays. The flights are numbered in the order in which they are scheduled.

The systemic allocation of delays to individual flights according to the order of scheduling is illustrated in Figure 2. The color-code identifies the degree of delay experienced by each flight. Flights in excess of the capacity in the first 15 minute period, spill-over into the second 15 minute period escalating the delays assigned to these flights. The spill-over cascades through the congested period (15 minute time slots 6 through 15) and into the period following the congested period (15 minute time slots 16 through 25).
Flights allocated slots within the available capacity. Note that the effect of scheduled over-utilization spills-over resulting in increasing delays to flights later in the schedule.

Delays also spill-over into period after the congested period.

Figure 2

The sum of the individual flight delays, known as the Total Delay Time, is a function of the degree of over-scheduling as well as the reservoir of capacity following the congested period. As shown in Figure 3, a queue of flights builds until the end of the congested period, at which time the reservoir of capacity allows the queue to dissipate.

Queueing of flights due to over-scheduling. The Total Delay time is the area under the Queueing function.

Figure 3
The Total Delay Time resulting from the scheduled utilization is defined by the equation:

\[
\text{Total Delay Time} = \frac{1}{2} \times (\text{Duration of Congested Period})^2 \times (\text{High Demand} - \text{Capacity}) \times \left[ 1 + \frac{\text{High Demand} - \text{Capacity}}{\text{Capacity} - \text{Low Demand}} \right]
\]

This equation is derived by calculating the area under the queue curve in Figure 3. The equation highlights the following properties:

1. **Conservation of Total Delays.** The Total Delay is independent of the order of the flights. The Total Delay is dependent on the relationship amongst the four terms: Capacity, High_Demand, Low_Demand and Congested_Period. Saying this another way, the only way to reduce the Total Delay is to remove flights.

2. **Duration of Congested Period is Critical.** The factor in the equation that has the biggest effect is the duration of the congested period (\(T_{\text{CongestedPeriod}}\)). This term is squared. For every additional unit of time in the congested period, the Total Delays increase geometrically.

3. **Reservoirs are Critical.** The Total Delays is not only dependent on the degree of over-scheduling, but also on the degree of under-scheduling after the congested period. The degree of under-scheduling provides a reservoir to absorb the spill-over from the congested period. A low degree of under-scheduling can result in extending the queue significantly.

The delays experienced by individual flights are shown in Figure 4. Flights early in the congested period experience relatively low delays. Flights at the end of the congested period and flights right after the congested period experience the highest delays.

(4) **Asymmetry of Individual Flight Delays.** The delays assigned to individual flights are a function of the location of the flight in the schedule. Flights
scheduled early in the congested period, are allocated less delays than those
flights later in the congested period.

3 EQUITY IN ALLOCATION OF OVER-SCHEDULED RESOURCES

Every society has rules for sharing goods and burdens among it’s members (Young, 1994). Some resources are managed through property rights and liabilities that are held and traded by private individuals or held by enterprises according to complex financial regulations. Other property rights are held by a governing entity and allocated according to societal needs. The mechanism for the distribution of property rights expresses the notions of equity in the division of the resources deemed reasonable by societal norms. The appropriateness of the equity is determined in part by principle and in part by precedent.

There are three main decisions that must be made in the allocation of commonly held property: (1) the supply decision determines the amount of resources to be distributed (e.g. arrival slots). (2) the distributive decision determines the principles and methods used to allocate the resources (e.g. first-scheduled/first-served), and (3) the reactive decision: determines the owners or users to the allocation scheme (e.g. slot substitutions and slot swapping). The focus of this paper is the implications of the distributive decision.

Varieties of Equity

Equitable allocation should be a simple process. Each party is allocated an equal distribution measured according to a single yardstick. The reality is that allocated resources are not equal. Claimant parties are in different situations and the agreement of single yardstick is difficult to achieve (Rae 1981). A wide range of philosophers (e.g. Aristotle, Maimonides) have examined the “combinatorics” of allocation of asymmetric resources to claimants using various yardsticks. These philosophers have developed appropriate allocation schemes for specific combinations. One of the emergent themes of these allocation schemes is that the equity formulas are usually based, either explicitly or implicitly, on a standard of comparison that ranks the various claimants on their relative desert (Young, 1994; pg 80).

Proportionality and Proportional Equity

One of the oldest and most widely used distributive principles is one that ranks claimants rights. This is the “Principle of Proportionality.” Proportionality is implicit in the mechanism of First-Come/First-Serve used in Air Traffic Control (ATC) and is the explicit in the mechanism of First-Scheduled/First-Served used in Traffic Flow Management (ATFM). This principle allocates the resources in proportion to the demand for the resource such that groups (e.g. airlines, passengers with specific demographics, or flights with specific emissive properties) will receive delays in proportion to their number of flights scheduled.

Proportional Equity is defined by the following equation:
Proportional Equity for Group (i) =

\[
\frac{\text{Total Delay for Group (i)}}{\text{Total Delay for all Groups}} \div \frac{\text{Number of Flights for Group (i)}}{\text{Total Flights for all Groups}}
\]

where i are groups of users (e.g. airlines) 1 through n.

The numerator represents the proportion of delays allocated to Group (i) with respect to the total delays allocated to all the Groups. The denominator represents the proportion of flights flown by Group (i).

When the proportion of delays is equivalent to the proportion of flights, the equity for the group is equal to 1. Proportional equity less than 1, implies that the group was allocated delays proportionately less than the number of flights scheduled. This group benefited from the allocation process. Proportional equity greater than 1, implies that the group was allocated delays proportionately more than the number of flights scheduled. This group was penalized by the allocation process.

*Proportional Equity for Individual Flights*

The equity of individual flights is entirely based on the magnitude of the spill-over of preceding flights. As a consequence, flights scheduled early in the congested period receive less that the average delay and have a proportional equity of less than 1. Flights scheduled late in the congested period, receive more delays and experience an equity of greater than 1. These results are illustrated in Figure 5. The vertical bars represent the delays allocated to individual flights (left y-axis). The magenta line defines the proportional equity for each flight (right y-axis).

*Proportional equity for individual flights is asymmetric. Flights are shown from first scheduled to last scheduled. The left y-axis identifies the delays experienced by each flight. The right y-axis identifies the proportional equity of each flight. Flights with proportional equity less than one experience an allocation advantage. Flights with proportional equity greater than one experience an allocation disadvantage.*

*Figure 5.*
Proportional Equity for Groups of Flights

Flights are scheduled by the individual airlines to meet the airlines network and connecting passenger and crew objectives and are constrained by the available airline resources. When flights are scheduled by independent airlines, without *a priori* knowledge of the schedules of other airlines, over-scheduling is feasible. Further, the independent scheduling results in an aggregate schedule that does not follow any rigid structure that could lead to symmetry in allocation of flight delays or equity. This schedule is effectively generated by a random process. Groupings of scheduled flights will experience proportional equity relative to the position of the flights in the schedule. A group of flights disproportionately positioned in the front (or back) of the congested period will experience less (or more) proportional equity.

The question posed by this model of over-scheduled resources is what is the probability that an aggregate schedule (without any structure imposed) will yield in proportional equity for any grouping?

4 PROBABILITY OF EQUITABLE ALLOCATIONS

The amount of delay allocated to individual flights is determined by position of the flights in the schedule and the effects of spill-over when demand is in excess of capacity. The asymmetric allocation of delays to individual flights constrains the opportunities for equitable allocation of proportional delays amongst groups of flights. Due to this phenomenon, proportional equity can only be achieved when the flights are scheduled according to a formula of rigid spacing that allocates low delay flights to each airline for every high delay allocation. When flights are scheduled independently by airlines (i.e. without knowledge of the schedule of other airlines), the cumulative schedule is effectively generated by a random process. The goal is to establish the likelihood of achieving an approximate proportional equity between groups in the schedule.

For every possible combination of sequence of flights in a schedule, their exists a percentage of groups of flights that will exhibit equitable allocation. *The probability of occurrence of these proportionally equitable schedules determines the likelihood of an equitable allocation.*

One way to determine the probability that a random allocation of slots is proportionally equitable is to enumerate all possible slot allocations and then to count the fraction of such allocations that are equitable. The combinatorial explosion in the number of possible schedules make this approach infeasible. An alternative approach is to estimate the probability of equitable allocations through Monte-Carlo computer simulation. An algorithm was developed for random generation of schedules and the subsequent determination of whether or not the slot assignment is equitable. Uniform distributions are used to ensure breadth of coverage. By repeating the algorithm multiple times using and counting the fraction of times that equitable allocations are generated, the probability of an equitable allocation can be estimated. [Note algorithm will be provided in full paper].
Figure 6 shows the results of simulation experiments to estimate the probabilities of proportional allocation of delays to randomly generated schedules. Each data point represents a combination of number of airlines (N) and number of flights per airline (M). The total number of flights in each schedule is N x M. One million randomly generated schedules were evaluated for each data point.

The x-axis represents the number of airlines N. The color-coded data-points represent the number of airlines (or the number of flights per airlines). The y-axis shows the probability of a proportionally equitable allocation for one million randomly generated schedules. The y-axis uses a logarithmic scale. The threshold for proportional equity was between 0.9 and 1.1. (i.e. +/-10% range).

These results indicate the elusiveness of individual groups (e.g. airlines, passenger demographics, aircraft/engine type) of randomly scheduling such that proportional equity can be achieved. The probability of obtaining an equitable allocation decreases both in the number of slots per airline and the number of airlines.

The probability decreases extremely fast in the number of airlines. Since every airline must have the same total delay, there are more constraints on the situation when there are more airlines. For example, if 3 airlines are involved and each airline has 4 slots, the probability of proportional equity is less than 1%. If 4 airlines are involved, the value drops to about .01%. If 5 or more airlines are involved, the probability is less than one in a million (no schedules with proportional equity were observed in the simulations). In fact, the only cases in which a “reasonable” probability of a proportional equity is
observed are cases involving 2 airlines, and this is still typically below 10% and closer to 1%.

Figure 7 further shows the dependence of results on the setting of the threshold for proportional equity for the particular case with 4 airlines and 5 slots per airline. The x-axis in this figure is the threshold for equity, expressed as a percentage of the proportional equity. For example, the value 4% means that the slot allocations are considered equitable if each group’s equity is within +/-4% of perfect equity (i.e. equity = 1). The probability of an equitable allocation is an increasing function of this percentage. The step-wise increases are due to the discrete nature of the slot assignments.

![Graph showing probability of fair allocation vs. allowable deviation]

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Dependence of results on the setting of the threshold for equitable allocation for the particular case with 4 airlines and 5 slots per airline. The x-axis in this figure is the threshold for equity, expressed as a percentage of the proportional equity. The probability of an equitable allocation is an increasing function of this percentage.

Figure 7

5 CONCLUSIONS & FUTURE WORK

Once a resource is over-scheduled, the damage is done. Except for some rare circumstances that impose a structure to the schedule, the probability of achieving an equitable allocation is very small (<10%).

This paper describes the feasibility of the equitable allocations of over-scheduled resources based on a mathematical model. The major results are as follows:

1. Asymmetry in Allocation – the natural dynamics of overscheduled resources is that flights later in the overscheduled time period will receive more delays
than those in the front of the queue. This is a property of the spill-over effects of over-schedules resources.

(2) **Collateral Damage** - Flights that are scheduled after the congested time period are also likely to receive (significant) delays due to spill-over effects.

(3) **Probabilities of Achieving Proportional Equity Amongst Users is Very Low** - equity for groups of flights (e.g. airlines, geographic access, environmental parameters), measured as the allocation of scarce resources in proportion to the number of flights in each group, can be achieved only by rigid spacing of flights in the schedule. Ignoring the requirements for airline networking (e.g. arrival and departure banks), the combinatorial probabilities for equitable allocation amongst groups of users is less than 10% when all users have the same number of flights. When the distribution of flights amongst users is not proportional, the probability of equitable allocation of scarce resources is much less than 10%.

(4) **Increasing Competition, Decreases Opportunities for Equity** – as the number of users increases (i.e. increased competition), the probability of scheduling with spacing to achieve equity decreases rapidly.

These results were generated by Monte Carlo simulation. Future work is underway to develop a closed-form solution to the problem. Also, future work is underway to examine the effects of non-homogeneous fleet mix, stochastic arrivals, non-compliance of slots allocations, the effect of cancelled flights, inter- and intra-airline swapping and compression.

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