Abstract—The saturation of resources National Airspace System (e.g., runways, fossil fuels, and water and air quality) obliges regulatory authorities to trade off performance and equity parameters to maximize the efficiency of the system as a whole. Researchers have conducted analysis of trade-offs using utility theory. The value of utility theory trade-offs is subject to: (i) the validity of the parameter weights used that must be derived by eliciting preferences of stakeholders, (ii) comparison of parameters measured in different units, as well as the relationships between the resources. Data Envelopment Analysis (DEA) overcomes these limitations and provides an assessment of the overall efficiency of the enterprises and enables management tradeoffs in use of resources in transparent and quantitative manner.

This paper describes the results of a trade-off of NAS performance and equity metrics for alternate Ground Delay Program (GDP) rationing rules. DEA analysis successfully (i) identified the composite magnitude of the difference in performance between the alternate rationing rules examined by computing the optimal parameter weightings for each rationing rule; and (ii) gave clear indication about where particular strengths lie for each of the rules as well as guidance on in which areas, and by how much, deficiencies exist through the use of a normalized efficiency score. The implications of these results are discussed.

Keywords- Data Envelopment Analysis; Equity; Rationing rules

I. INTRODUCTION

The growth in demand for air transportation in the U.S. has resulted in saturation of the capacity of many key resources in the NAS including runways, gates, and airspace corridors [1]. The resulting growth in the number of flights and destinations served has also placed pressure on several natural resources such as fossil fuels, air and water quality, and environments free of excessive noise [2]. Growth in demand for air transportation and the associated flights is forecast to continue for next several decades [3].

In the presence of saturated resource capacity, regulatory authorities are obliged to restrict the use of NAS-wide resources to ensure safety and to allocate the use of the resources to eligible parties. Traditional methods of allocation of resources for runway and airspace slots (e.g. Ground Delay Program and Airspace Flow Program) have been designed to equitably minimize the flight delay allocated to each user [4]. Future allocation schemes, however, will be obliged to trade off the performance and equity interests of multiple stakeholders such as passenger trip delays, airline flight delays, airline fuel consumption, airline delay equity, airline fuel consumption equity, and passenger geographic access equity [5] [6].

The trade-off of performance and equity parameters can be conducted using utility theory. This approach has two shortcomings. The first is that it relies on the assignment of weights to each parameter in the trade-off and the assignment of these weights is generally highly subjective. Typically, these weights are derived from surveys or elicitation by subject-matter-experts.

The second shortcoming of utility theory approaches is that comparison of parameters requires that they be normalized to a common set of units such as cost. Normalizing parameters in a context such as equity is often not possible. For example, the distribution of delays between hub, non-hub, and rural origins does not have a natural conversion into costs.

An alternative method to compare parameters in different dimensions is Data Envelopment Analysis (DEA). DEA is a mathematical programming technique for determining the efficiency of entities or systems across multiple performance and equity measures by comparison against their peers and the best-in-class Pareto frontier.

This paper demonstrates the results of a trade-off analysis of NAS performance and equity metrics for alternate Ground Delay Program (GDP) rationing rules proposed by Manley and Sherry [5]. In addition to the traditional measures of flight delays, these alternate rationing rules were designed to account for performance metrics such as passenger trip delays and fuel consumption, and equity metrics such as airline flight delay equity, airline fuel consumption equity, and passenger geographic access equity. Utility theory analysis of these results proved inconclusive due to the coupled nature of the metrics.

The DEA analysis identified that the Ration-by-Passenger rule is the most efficient of the different rules examined, as it significantly outperforms the baseline Ration-by-Schedule rule along with all other alternative rules proposed. The Ration-by-
Fuel-Flow – low precedence rule performed the worst in the aggregate DEA analysis.

These results were arrived at by the computation of a normalized efficiency score for each of the rationing rules under the assumption that optimal resource allocation decisions are made to achieve the best possible performance given the constraints of the rationing rule. On that basis, the DEA model computes the most favorable weightings for each of the rationing rules for the comparison.

In addition to the “stack-ranking” of the different rationing rules, the DEA results also give a clear indication about where particular strengths lie for each of the rules (e.g. overall fuel burn difference and passenger delay difference are the advantages for Ration-by-Passenger) along with guidance on which areas deficiencies still exist (e.g. airline fuel burn inequity for the Ration-by-Passenger rule) and how large those normalized performance gaps are to the best performing rules that make up the Pareto frontier.

The successful application of DEA to this form of analysis provides regulators and decision-makers a powerful tool to evaluate complicated trade-offs in parameters measured in different units.

The paper is organized as follows: Section 2 provides an overview of the alternate rationing rules evaluated by Manley and Sherry [5]. Section 3 describes the DEA methodology. Section 4 describes the results of the DEA analysis. The conclusions and implications of these results are provided in Section 5.

II. RATIONING DELAYS AND NAS PERFORMANCE AND EQUITY

The National Airspace System (NAS) supply the infrastructure and resources to provide air transportation service to the nation. Because the NAS is publicly owned and operated, the infrastructure and resources must be regulated in a manner that reflects the social, political, economic, and security interests of the citizens of the nation.

Since airlines are the dominant users of the NAS, airline performance is also the primary measure of NAS performance. Airline flight performance is published by the Bureau of Transportation Statistics (BTS) [7]. The BTS reports on-time performance. Recent updates to the BTS also include diverted and cancelled flights.

When flight demand is in excess of capacity, the FAA regulates arrival demand at constrained airports and airspace though the Traffic Flow Management Initiatives such as the Ground Delay Program (GDP), Airspace Flow Program (AFP) and Miles-In-Trail (MIT). When these programs are designed, special care is taken to ensure equitable allocation of delays between airlines. The traditional measure of equity is the proportional delay equity computed as follows:

Equity for Airline (i) =

\[
\frac{[\text{Total Flight Delay for Airline (i)} \times \text{Total Flight Delay for all Airlines}]}{[\text{Number or Flights for Airline (i)} \times \text{Total Number of Flights for all Airlines}]}
\] (1)

Equity values lower than one indicate that the airline was granted a lower proportion of flight delays given the proportion of flights flown. Equity values greater than one indicate that the airline received a higher proportion of delays than the proportion of flights flown.

Manley and Sherry [5] proposed a method for evaluating the overall equity of the allocation by summing the differences between an airline’s equity and perfect equity (i.e. 1). To enable this cumulative assessment, the individual equity was converted to a negative logarithm of the quotient of (i) ratio of an airline’s total flight delay over the total flight delay for all airlines, (ii) ratio of the number of the airline’s flights over all the airlines flights. Using this metric, "perfect equity" is represented as 0. If the airline’s equity is positive, the airline is assigned less delay than its fair share. Conversely, if the airline’s equity is negative, then the airline is assigned more delay than its fair share.

Researchers have also investigated the impact of NAS decisions on consumers and other stakeholders.

- Passenger Trip Delays: The delays experienced by passengers have been computed by Bratu and Barnhart [8] and Wang and Sherry [9]. Passenger trip delay is the difference in time between the Scheduled Arrival Time of the passenger and the Actual Arrival Time of the passenger. Differences are the result of delayed flights, rebooking due to cancelled flights, rebooking due to oversold flights and diverted flights. Studies have shown that although only 2% of passengers are impacted by cancelled flights, the delays experienced by these passengers account for 40% of the total passenger trip delays experienced by all passengers.

- Excess Fuel Burn: The Excess Fuel Burn indicates the quantity of excess fuel burn due to delays. This metric computed the additional fuel consumed by engines or auxiliary power units during delays. See Manley and Sherry [5].

- Inter-Airline Excess Fuel Burn Equity: This measure evaluates the degree to which the excess fuel burn experienced by flights regulated with delays is equitable between airlines. See Manley and Sherry [5].

- Passenger Geographic Equity: This measure evaluates the degree to which passenger trip delays and/or flight delays are equitable between geographic regions and demographics. Do larger metropolitan areas get as much delays as smaller metropolitan areas and rural areas?

Manley and Sherry [5] examined the performance and equity for six alternate GDP rationing rules that were applied for 197 GDPs at Newark International Airport (EWR) in 2007:

- Ration-by-Schedule (RBS)
- Ration-by-Passengers (RBPax)
- Ration-by-Aircraft Size (RBAcSize)
- Ration-by-Distance (RBD)
• Ration-by-Fuel Flow - high precedence (RBFFh)
• Ration-by-Fuel Flow - low precedence (RBFFl)

The results indicated that there exists a trade-off between GDP performance and GDP equity. When the objective of the rationing rule is to maximize performance, Ration-by-Passengers is the best rule to implement at EWR. This rule maximizes passenger throughput to the GDP airport and results in a 23% decrease in passenger delays and a 57% decrease in excess surface fuel burn compared to Ration-by-Schedule (the current rule) with no change in total flight delay. On the other hand, when the objective of the rationing rule is to minimize inequity among airlines and passengers, the Ration-by-Schedule is the best rule to implement at EWR.

The application of utility theory to examine the trade-off between performance and equity in the dimensions of flight delays, excess fuel burn, passenger trip delay, and passenger geographic access was inconclusive. The results of the utility theory approach were limited by:

(i) The validity of the parameter weights used that must be derived by eliciting preferences of stakeholders.
(ii) Comparison of parameters measured in different units.

Data Envelopment Analysis (DEA) overcomes these limitations and provides an assessment of the overall efficiency of enterprises assuming that those tradeoffs are made as part of a conscious management decision. The DEA methodology and the results of analysis of the alternate rationing rules are described in the next two sections.

### III. DEA METHODOLOGY

In measuring and comparing the performance of different entities, four essential steps can be identified:

1. Identification of appropriate metrics that will be true indicators of good or bad performance.
2. Collection of performance data.
3. Determination of which methodology to use for comparing performance.

Proper application of these steps will ensure a performance comparison that is meaningful and accurate. The following sub-sections describe the details of each of these steps for this analysis of delay equity rules.

#### A. Identification of Metrics

The first step to determining the right performance metrics is to identify who the stakeholders of the analysis are. Only once the stakeholders have been clearly determined can one discuss which performance measures to use since different stakeholders will have a different view of what the goal or meaning of the enterprise is.

In this analysis, we rely on the identification of stakeholders and determination of performance metrics carried out in Manley and Sherry [5]. In that analysis, stakeholders were defined as not only individual airlines but also the traveling public, and as such performance metrics were defined to include both fuel burn and passenger delay metrics. Furthermore, the view was taken that performance measures shouldn’t simply be optimized at the global level (e.g. by minimizing total delay minutes) but also that any such optimization should take into account the equity of the impact on individual airlines and individual passengers.

Having said this, it is crucial to note that if this analysis had not already been performed by Manley and Sherry, the determination of the appropriate performance metrics would have been a highly important step to include in the present analysis. We are stressing this fact since analyzing overall system goals and determining the right performance metrics is an area that is far too often overlooked by analysts looking to compare the performance of different entities.

#### B. Collection of Performance Data

The performance data for the study was collected by Manley and Sherry [5] for each of the rationing rules listed in the previous section and is presented in Figure 1 and Figure 2.
C. Determination of Methodology

Data Envelopment Analysis (DEA) provides a method for assessing the efficiency of enterprises, so-called Decision-Making Units, or DMUs, in converting inputs (consumption of resources such as staff, raw materials, etc.) into desirable outputs (production of widgets, throughput of aircraft, etc.). DEA ranks the DMUs in the study relative to one another, with the most efficient receiving a “score” of 1.0. The most favorable weights are assigned to inputs and outputs for each DMU, reflecting the assumption that management make intentional tradeoffs between the set of outputs to be maximized and the set of resources to be consumed.

A mathematical introduction to DEA follows. This is the first, most basic DEA model as proposed by Cooper, Charnes, and Rhodes [10] in 1979, giving it its name, CCR. The objective is to identify the DMU(s) with the best inherent efficiency in converting inputs \( x_1, x_2, \ldots, x_n \) into outputs \( y_1, y_2, \ldots, y_m \). We then want to rank all other DMUs relative to the most efficient DMU(s).
We then convert this initial program into a linear program by setting the denominator in the objective function equal to 1 and moving it to the constraints section, as well as cross-multiplying and rearranging the original condition, resulting in the following modified problem.

Finally, the program is converted to its dual and slacks are added.

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In this study, each delay rationing rule is considered one DMU. Since all of the metrics identified should be minimized we considered them as inputs to the process. Also, all of the DMUs produce exactly the same number of desirable outputs in the form of aircraft and passengers moved. Consequently, for simplicity’s sake, we create an artificial output and set it to the constant 1 for all DMUs to reflect this fact.

In conducting the analysis, several issues had to be addressed given the present dataset. The first such issue was the ratio of inputs and outputs to the number of DMUs in the analysis. In a situation with too many inputs and outputs relative to the number of DMUs, the basic DEA algorithm will permit most DMUs to appear efficient given the flexibility of the DEA algorithm in assigning weights to different inputs and outputs. Dyson et al. [11] suggest that the number of DMUs should at a minimum be equal to \(2n \times m\), where \(n\) is the number of inputs and \(m\) is the number of outputs. That means that in this analysis, the minimum number of DMUs should be 8, but only 6 DMUs are present.

In order to address this deficiency, previous studies have introduced the idea of an artificial super-efficient DMU (see for example Basargan and Vasigh [12]). This is an artificial DMU that is assigned the best values present in the study for each input and output, and this approach allows for better discrimination among all other DMUs.

However, using this approach with CCR methodology results in all DMUs with one or more input value that is the best in the comparative dataset will still receive a fully efficient ranking since the algorithm will simply assign all weights for those inputs for each DMU. Using the performance data in this study, three of the six DMUs received a fully efficient score when using this approach, which made it unsuitable as it did not provide enough discrimination among the DMUs.

Hence, an alternative measure of efficiency was considered, the Slacks-Based Measure (SBM) of efficiency, as introduced by Kone [13]. The SBM algorithm is very similar to the CCR methodology, with the difference that its objective function is based on the slack variable measures. Here, the objective is to minimize the amount of slack; all other variables in the objective function are constants.

Using the SBM algorithm with the addition of a super-efficient DMU, the analysis produced effective discrimination among the DMUs.

The SBM scores were calculated using the Excel-based DEA-Solver tool.

IV. COMPUTATION OF PERFORMANCE COMPARISON SCORES AND ANALYSIS

The resulting SBM scores are presented in Figure 3, and indicate the strongest performance for the Ration-by-Passengers rationing scheme and the worst performance for Ration-by-Fuel Flow – low precedence.
To understand what is driving these efficiency scores and what would be required for further improvements one must examine the values of the slack variables, as shown in Figure 4.

The tables in Figure 4 include a “projection” value, which is the performance target necessary for each DMU to be considered fully efficient for each performance parameter. In order to achieve full efficiency, all of these performance gaps must be closed.

Examining the data in Figure 4, we can see that Ration-by-Passengers achieves its strong performance score primarily through its fuel burn difference and passenger delay difference values. Its primary shortcoming is its poor performance in terms of airline inequity. Conversely, Ration-by-Fuel Flow - low precedence performs poorly on all parameters except for passenger inequity, where it has achieved the best performance among the different DMUs considered.

In order for any one of these rationing rules to achieve full efficiency, all of these performance gaps would have to be closed. However, then the important question becomes: Is it possible to close these performance gaps without adversely impacting the other performance parameters?
The answer to this question comes from a simple correlation analysis of the inputs to the analysis summarized in Table 1. The correlation analysis indicates that overall efficiency gains come at the expense of individual equity.

At the 0.10 significance level, there is a negative correlation between inter-airline inequity and fuel burn difference. This means that as the inter-airline equity measure improves, total fuel burn goes up. So for example, if Ration-by-Passengers or Ration-by-Fuel Flow - high precedence were to attempt to improve the inter-airline equity measure which is new each of the two rules have their most significant deficiencies, it should be expected that total fuel burn – where both rules currently perform well – would worsen. This reflects the inherent conflicts that exist in some parts of the air transportation system; in this instance, overall system efficiency could be achieved at the expense of fairness to individual airlines.

However, inherent positive effects also exist in the system: The correlation analysis shows that as overall fuel burn is reduced, so is the number of total passenger delay minutes.

<table>
<thead>
<tr>
<th></th>
<th>Pearson Geographic Access Inequity</th>
<th>Fuel Burn Difference</th>
<th>Passenger Delay Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inter-Airline Inequity</td>
<td>Pearson Correlation Sig. (2-tailed)</td>
<td>0.63</td>
<td>-0.94</td>
</tr>
<tr>
<td></td>
<td>Sig. (2-tailed)</td>
<td>0.18</td>
<td>0.00</td>
</tr>
<tr>
<td>Passenger Geographic Access Inequity</td>
<td>Pearson Correlation Sig. (2-tailed)</td>
<td>-0.66</td>
<td>0.15</td>
</tr>
<tr>
<td>Fuel Burn Difference</td>
<td>Pearson Correlation Sig. (2-tailed)</td>
<td>0.76</td>
<td>0.08</td>
</tr>
<tr>
<td>Passenger Delay Difference</td>
<td>Pearson Correlation Sig. (2-tailed)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE I. CORRELATION ANALYSIS OF INPUTS

V. CONCLUSIONS

This analysis shows that the DEA approach works for handling a real-life multi-dimensional problem with inputs in different units. It gives decision-makers the insight necessary to make informed decisions about the tradeoffs between sometimes conflicting objectives.

The DEA analysis shows that the Ration-by-Passengers rule is the most efficient of the different rules examined, significantly outperforming the baseline Ration-by-Schedule rule. The DEA results also give a clear indication about where particular strengths lie for each of the rules (overall fuel burn difference and passenger delay difference are the advantages for Ration-by-Passengers) as well as guidance on in which areas, and by how much, deficiencies still exist (airline fuel burn inequity for Ration-by-Passengers).

The methodology is also useful for uncovering the meaning of the results - improved “systemic” efficiency in terms of global fuel burn and passenger delay differences come at the expense of equity among airlines and passengers. This is where the main tradeoffs must occur.

The performance analysis examined highlights the importance of a comprehensive measure of performance efficiency but also illustrates the technical challenges in effectively computing such a measure. A deep understanding of the nuances of the DEA technique is necessary in order to achieve accurate and meaningful comparative measures of performance. Future work will include methodological research to deepen understanding of how to most appropriately apply DEA in an aviation context, as well as further applications of DEA on other aspects of the Air Transportation System, such as airports.

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**AUTHOR BIOGRAPHY**

**David Schaar** is currently a Ph.D. student at the Center for Air Transportation Systems Research in the department of Systems Engineering and Operations Research at George Mason University’s Volgenau School of Information Technology and Engineering. He has a Master of Science degree in Industrial Engineering and Management from the Linköping Institute of Technology in Sweden. He is currently employed as a Project Manager with the Corporate Executive Board of Arlington, VA. His research interests include performance measurement and benchmarking in aviation.

**Lance Sherry** Dr. Sherry is Associate Professor-Research in the Dept of System Engineering and Operations Research and serves as the Executive-Director for the Center for Air Transportation Systems Research (CATSR) at George Mason University. Dr. Sherry is a system engineer and analyst with over 25 years of practical experience in air transportation operations, strategic planning, benefit/cost analysis and technology innovation and transfer. Dr. Sherry has a B.Sc. in Electrical Engineering from Brown University and a Ph.D. in Industrial and Systems Engineering from Arizona State University.