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STATISTICAL SEPARATION STANDARDS FOR THE AIRCRAFT-APPROACH PROCESS

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Abstract

Demand for air transportation has been increasing. A response to this is enhancing the runway utilization and throughput before investing on new runways or instruments. Runway throughput can be increased by reducing in-trail landing separation between aircraft, but the consequence may be an increase in the chance of a severe wake vortex encounter or a simultaneous runway occupancy (or go-around). Current instrument flight rule (IFR) standards provide fixed separation minima for given pairs of wake vortex weight classes of aircraft. In practice, the observed separation is a random variable and fluctuates near or above the specified minimum. In this paper, we propose a framework for *statistical* separation standards that specifies not only a lower bound for the separation but also a standard for the *target value* and the *variance* of the process. We address the question of what a more efficient separation standard is in order to control the risk in the approach process. We also consider whether separation variability may be reduced by employing such standards, i.e. more detailed standards that take the "realized variability" into account. Analytical results of this study suggest that (under specific assumptions) throughput can be increased to some extent without degrading safety for the given facilities, infrastructure, and weather condition. The arguments and concepts are illustrated with statistical observations from Detroit airport (DTW).

1 Introduction

Demand for air transportation has been increasing and will continue to rise (see [1], for example). A response to this is enhancing the runway utilization and throughput before investing on new runways or instruments. However, *risk*, as the probability of loss of safety, is the other side of the *throughput* coin. Runway throughput can be increased by reducing in-trail landing separation between aircraft, but the consequence may be an increase in the chance of a severe wake vortex encounter or a simultaneous runway occupancy (or go-around).

Although air traffic control (sequencing and separation) is safe, there are some inherent risks in their operations. The system must be controlled so that the probability of a severe event, such as a fatal accident, is extremely rare. In general, this is possible since many things must go wrong for such an event to occur. But to achieve this, these rare events (and their associated precursors, such as simultaneous runway occupancy) should be controlled so that their probability remains below a very small acceptable limit. Designing such a robust control regime is a challenge because of the high complexity and variability of the air transportation system.

Controlling this complex system cannot be achieved just via in-operation/in-action feedback control. A pro-active strategic control approach is imperative. Pro-active control is achieved through well designed plans. Different types of plans include goals, objectives, strategies, tactics, rules, regulations, policies, procedures, etc [2]. In air traffic control, these plans include efficient separation standards and procedures of approach to the runway, for example. The focus of this paper is a new methodology for setting these separation standards.

This paper addresses the performance of the approach process as an important limiting component in the air transportation system. Several empirical studies have analyzed statistics of the landing process, including runway occupancy time and landing time interval (see [3]-[9]). This research utilizes data obtained from the multilateration surveillance system of Detroit Metropolitan Wayne County (DTW) airport, analyzed by Jeddi et al. [9]. The objective in this paper is to reduce the probability that a following aircraft reaches the runway threshold before the leading aircraft exits the runway. Such an event results in simultaneous runway occupancy (SRO) – or requires a costly go-around/missed-approach procedure [10] as mitigation. In this paper, we consider statistical separation standards only with respect to SRO risk and do not consider wake vortex risk – that is, we assume all operations are absolutely safe from wake encounters.

The paper is organized as following. Section 2 describes a process control oriented approach to analyze the approach phase. Section 3 formulates a mathematical separation model, and introduces a statistical approach to design a separation standard. A numerical example is provided in this section. Section 4 presents conclusions and proposals for future research topics. A list of acronyms is given in Appendix I.

2 The Final Approach Process

To analyze the behavior of a system it is necessary to obtain an overall view about its operational structure. A process oriented approach provides such a view and this paper models air traffic control operations from this perspective.

The following notations are used throughout the paper.

Notation

$LTI_{k,k+1}$ landing time interval between aircraft *k* and *k*+1 measured at the runway threshold

$IAD_{k,k+1}$ inter arrival distance of aircraft *k* and *k*+1 when aircraft *k* is over the threshold

ROT_k runway occupancy time of aircraft *k*

MS minimum separation

LB lower bound

S_j separation distance between an aircraft pair controlled by controller *j*

Δ_j constant buffer added to *MS* by controller *j*

TV_j target value of controller *j* which is $MS+\Delta_j$

TV_t time-based spacing target value

TV_n nmi-based spacing target value

p_j proportion of aircraft pairs guided by controller *j*

ε_j separation error from TV_j of controller *j* assumed to have $N(0, \sigma^2_j)$ distribution

σ_j standard deviation of the imposed control by controller *j*

α_R acceptable probability for $LTI_{k,k+1}<ROT_k$

α_{WV} acceptable probability for moderate or severe wake vortex encounter

μ_{LTI} mean of *LTI* probability distribution

τ_{LTI} mode of *LTI* probability distribution which is TV_t

The indices are dropped whenever the general parameter or variable is the subject. A variable with a '*' is an optimal value.

2.1 Air Traffic Control Processes

We consider the air traffic control system as a process with inputs and outputs (Figure 1). System outputs include throughput, risk, safety, delays, and so forth. These are the result of various inputs to the system. We cluster the inputs in three categories:

- Operational human participants: pilots, air traffic controllers, and other people involved in the operational decision making process.
- Mid-term plans: standards such as separation standards, procedures such as landing/departure procedures, rules, training curriculum of pilots and air traffic controllers, decision support systems such as statistical analysis tools designed to give feedback to managers, etc. This cluster of inputs may be revised or changed by high level decision makers in medium terms, for example 3 years.
- Uncontrollable or long-run controllable factors: these are factors for which decision makers have no control in the short-run or have no control at all. They include weather, supply and demand interactions in the market, or long-run changes in aircraft technology, airports, navigation systems, number of runways, etc.

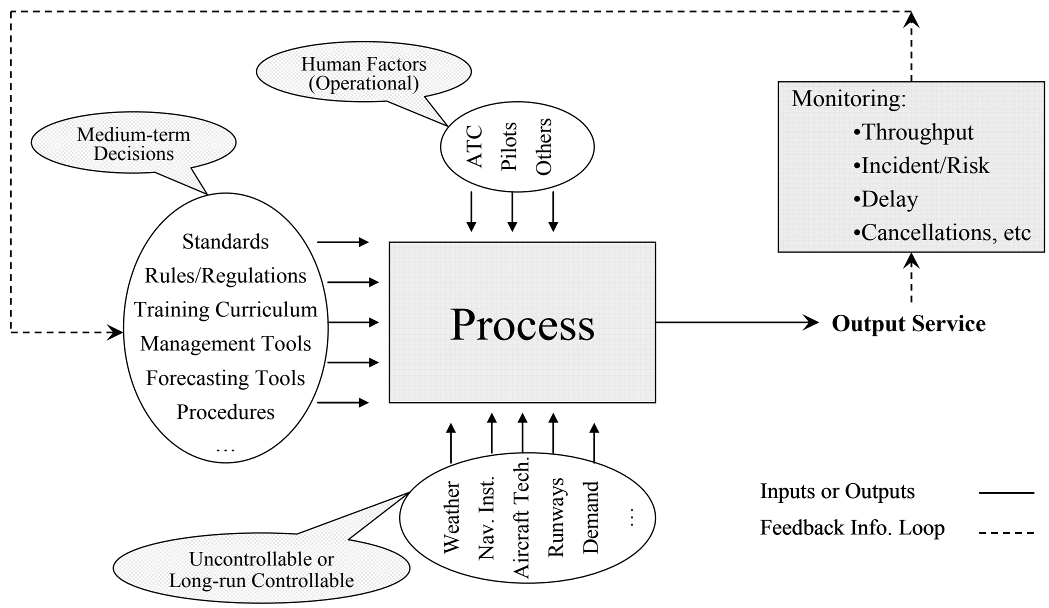

Figure 1. A Feedback Loop on an Air Transportation Process

Output characteristics of the process such as throughput, runway utilization, frequency of incidents, and frequency/length of delays are random variables because of uncertainty involved in the first and third (and sometimes in the second) categories of inputs. Attempts should be made in probabilistic operations to reduce the variance as much as possible. However, there is always some inherent and uncontrollable variation in the process, which can not be completely eliminated.

Statistical Process Control of the Approach Process

Concepts of variability and control are widely developed and employed in the manufacturing industry (see [11] and [12], for example). In the context of the quality of manufactured items, the natural inherent variability is often referred to as a *stable system* (or *pattern) of chance causes*, and is viewed as an acceptable source of variation. However, any variation in excess of this natural pattern is unacceptable and its detection and possible correction is required. Variations outside the stable pattern are known as *assignable causes* of quality variation. A process which operates in the absence of any assignable causes of erratic fluctuations is said to be *in statistical control*.

We employ the general knowledge of statistics and adopt statistical tools used to monitor stochastic processes in manufacturing, named *statistical process control*, to the approach/landing process. This research introduces a methodology for planning, analyzing, and controlling operations of the approach process, first, to ensure that separation standards will not be violated as a result of high landing demand, and secondly, to reduce output variability to possibly gain a throughput increase.

We assume that the system *capacity* is fixed for a given weather condition and runway and airport infrastructure. Therefore, the *utilization* of a given system/capacity can be improved, but not the capacity itself. We use the term *throughput* to represent the realized number of planes landing per unit of time.

2.2 Landing Risks in the Final Approach and Runway

For a given infrastructure (quantity and quality of physical equipment and airport facilities), aircraft capabilities, and weather conditions, the main limiting factor of throughput is the separation spacing between aircraft in the approach phase. This research addresses time and distance separation in the approach phase for a single runway or parallel independent runways.

Separation of an aircraft pair is a random variable due to the nature of the process inputs and components. Figure 2 shows a final approach process and notional probability distribution functions of *LTI* and *ROT* (for examples of these distributions see [3]-[9]).

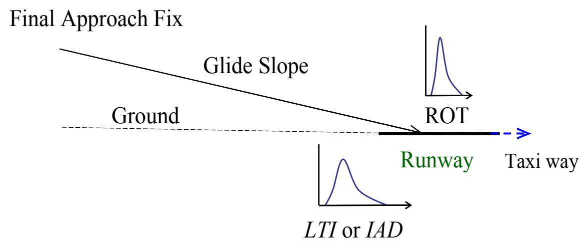

Figure 2. A typical Final Approach Process

We categorize pairs of aircraft into four classes of 3.0, 4.0, 5.0, and 6.0 nmi based on the separation standard of Table 1 [13]-[14]. (One exception, which is considered for some runways, is a separation standard of 2.5 nmi for the follow-lead pairs of small-small, large-small, and large-large aircraft.) The *LTI* probability distribution functions (PDF's) for the individual classes may be different, so we consider them separately in analyzing separation standards. The realized PDF of *LTI* for *all* aircraft is the mixture of the individual class-PDF's, where the class-PDF's are weighted by the fraction of landings under each separation class.

The separation spacing is necessary for two reasons. First, the spacing should be large enough to avoid a simultaneous runway occupancy or runway incursion. Second, the spacing should be large enough to diminish the risk associated with the wake vortex of the leading aircraft.

Table 1. IFR Approach Threshold Separation Minima (nmi)

Following Aircraft	Leading Aircraft			
	Small	**Large**	**B757**	**Heavy**
Small	3	4	5	6
Large	3	3	4	5
B757	3	3	4	5
Heavy	3	3	4	4

Under IFR, the air traffic controller (ATC) is responsible to guide aircraft/pilots to assure the separation minima. It is assumed that if these minima are strictly respected, no aircraft cross the threshold to land while the lead aircraft is still on the runway. Thus, it is assumed that $P\{LTI < ROT\} = 0$ where *LTI* is measured at the runway threshold.

ROT depends on factors such as the number and structure of exit-ways, touchdown speed, wind velocity along the runway, and aircraft breaking capability. These factors can not be changed easily. So we assume a given PDF for *ROT* and study the possible changes in the PDF of *LTI* and *IAD*.

In peak periods, there is a high landing demand for runways. In such cases, ATC and pilots push aircraft as close as possible to each other to satisfy the arrival demand. This increases the probability of violating the separation minima.

Considering that the arrival demand frequently exceeds capacity (for a period of time), this paper addresses the following questions:

1) What is a mechanism to control and reduce the likelihood of *separation minima violations*, *serious wake vortex encounters* and *runway incursions* or SRO's?
2) Can throughput be increased for a given infrastructure while maintaining the risks below a specific level?

To address the first question, we investigate a different conceptual perspective in the design of separation standards, which is addressed in the next section. We show that the proposed new separation standards may lead to smaller separation variability. This means that (on average) aircraft can fly/land more closely to each other without increasing the related risks; this addresses the second question.

3 Statistical Separation Standards

An effective separation design shall address two challenges. First it shall control the risk of severe wake vortex encounter and simultaneous runway occupancy (or go-around) under peak operations. Secondly it shall possibly reduce inherent variability of the separation spacing which may result in increasing the throughput without increasing the risk.

Conceptually, this study models the separation as a random variable and analyzes the nature of its variance. Then we propose an algorithm to set separation standards based on statistical tools which posses two aforementioned characteristics.

3.1 Modeling the Separation

Under IFR the ATC is to maximize landing throughput subject to the spacing minima from Table 1, for different pair types of aircraft. Index controllers by j=1,…,m. To respect a given *minimum separation* (*MS*), the controller j needs to add a *buffer spacing* Δ_j to the *MS* while pushing as close as possible to the *MS* in order to maximize the throughput. This concept is also suggested in [15]. Adding the buffer Δ_j means that the controller targets a separation value of $TV_j=MS+\Delta_j$, as illustrated in Figure 3. In this figure, the leading aircraft is at point 0 and the following aircraft is somewhere on the *IAD* axis. We name the buffered-spacing as *target separation value* or for simplicity *target value TV*. In this paper, we assume that ATC j decides on his/her TV_j independently based on his/her experience, cognitive tendency, and level of risk aversion. Thus, TV_j may differ between air traffic controllers.

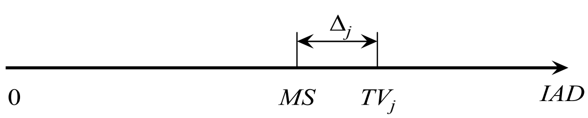

Figure 3. A Buffer Spacing Added by ATC *j* to the *MS*

Assume that a pool of infinitely many pairs of aircraft of a given type, e.g., Large-Large, is lined up to land on a single runway which is guided by m controllers. A pair of aircraft from this pool which is controlled by ATC j has a separation S_j which is a random variable. Then

$$S_j = MS + \Delta_j + \varepsilon_j \quad \text{for } j = 1,\dots,m, \tag{1}$$

where ε_j represents the random separation error of ATC j around his/her target separation. We assume $N(0, \sigma_j^2)$ distribution for the error terms ε_j.

Let p_j be the proportion of the aircraft from this pool which is controlled by ATC j. The overall separation S of the pool is a mix of separations S_j with proportions p_j, j=1,…,m. We can write

$$S = TV + \varepsilon = MS + \Delta + \varepsilon \tag{2}$$

where

$$\begin{aligned} &\Delta = \Delta_j \quad w.p. \quad p_j \text{ for } j = 1,\dots, m \\ &\varepsilon = \varepsilon_j \quad w.p. \quad p_j \text{ for } j = 1,\dots, m \\ &\textstyle\sum_{j=1}^{m} p_j = 1. \end{aligned} \tag{3}$$

Assuming $\varepsilon_j \sim N(0, \sigma_j^2)$ results in a normal distribution for the separations S_j and S in equations (1) and (2). A normal distribution is also independently suggested by Vandevenne and Lippert [16], Xie et al. [17], and Xie [6]. Haest and Goverts [18] address the contribution of radar plot accuracy to horizontal separation standards with the normality assumption for displayed polar coordinates on the radar screen.

From (2) and (3), the expected value of S is:

$$\begin{aligned} E(S) &= E(MS) + E(\Delta) + E(\varepsilon) \\ &= MS + \textstyle\sum_j p_j \Delta_j + \sum_j p_j E(\varepsilon_j) \\ &= MS + \textstyle\sum_j p_j \Delta_j, \end{aligned} \tag{4}$$

since MS and Δ_j are constants, and $E(\varepsilon_j)=0$. Also, because $E(\varepsilon_j^2)=\sigma_j^2$, the variance of S is

$$\begin{aligned} Var(S) &= Var(MS) + Var(\Delta + \varepsilon) \\ &= Var(\Delta) + Var(\varepsilon) + 2Cov(\Delta, \varepsilon) \\ &= Var(\Delta) + \left[E(\varepsilon^2) - E^2(\varepsilon)\right] \\ &\quad + 2\left[E(\Delta \cdot \varepsilon) - E(\Delta)E(\varepsilon)\right] \\ &= Var(\Delta) + \textstyle\sum_j p_j \sigma_j^2, \end{aligned} \tag{5}$$

where Var(MS) is zero and the last equality follows by conditioning on j and using $E(\varepsilon_j)=0$. Now, if we fix $\Delta_j=\Delta$ – equivalently, if we fix $TV_j = TV$ (that is, if each controller uses the same target spacing) – the variation caused by the variable Δ is eliminated. Thus, Var(Δ) is zero, and a new separation S' is obtained with a smaller variance

$$Var(S') = \textstyle\sum_j p_j \sigma_j^2. \tag{6}$$

As a result, it is expected that the *target-value based separation standard* provide a smaller overall variability for any given landing aircraft pair.

For the case of two ATC, (5) can be written as

$$Var(S) = p_1(1-p_1)(\Delta_1-\Delta_2)^2 + p_1\sigma_1^2 + (1-p_1)\sigma_2^2 \,. \qquad (7)$$

By making $\Delta_1=\Delta_2$, the separation variance decreases by the following proportion:

$$\frac{p_1(1-p_1)(\Delta_1-\Delta_2)^2}{p_1(1-p_1)(\Delta_1-\Delta_2)^2 + p_1\sigma_1^2 + (1-p_1)\sigma_2^2}\,.$$

The following numerical example illustrates these concepts.

Example

Suppose that two groups of ATC guide landings on a runway under IFR with proportions $p_1=p_2=0.5$ and with equal error variances $\sigma_1^2=\sigma_2^2=\sigma^2$. Table 2 shows the variance reduction percentage for the assumed scenarios. The first column is the assumed overall variance of aircraft separation, Var(S)=0.45^2, which is assumed fixed for all scenarios. The second column $|\Delta_1-\Delta_2|$ is the assumed absolute difference between buffer spacings of the two ATC groups. The third column $\mathrm{Var}(S')$ is the overall variance of aircraft separation *if the buffer spacings were made equal*: $\Delta_1=\Delta_2$. The last column is the percent reduction in variance. For example, in the third scenario, changing the difference in buffer spacing from $|\Delta_1-\Delta_2|=0.4$ to $|\Delta_1-\Delta_2|=0$ results in a 20% reduction in the variance of aircraft separation. The values for $\mathrm{Var}(S')$ in the third column were computed by substituting $Var(S') = p_1\sigma_1^2 + p_2\sigma_2^2$ from (6) into (7), giving:

$$Var(S') = Var(S) - p(1-p)(\Delta_1-\Delta_2)^2\,. \qquad (8)$$

■

Table 2. Scenarios of the Variance Reduction

Var(S)	$\|\Delta_1-\Delta_2\|$	$\mathrm{Var}(S')$	% Var. reduction
0.45^2	0.2	0.44^2	5
0.45^2	0.3	0.42^2	11
0.45^2	0.4	0.40^2	20
0.45^2	0.5	0.37^2	31

In this analysis, we do not include separation effects due to gaps in the schedule or arrival process since the effect of the controller influence is the subject of the study. In particular, equation (2) assumes that the aircraft are lined up to land one after another, so we consider no term in the equation representing a possible gap waiting for the next plane to arrive at the terminal airspace. In practice, the aircraft may not be readily present in the line, so some longer separations would be realized that cannot be explained by the normal distribution; this is also suggested by [6], [16], and [17]. So, skew-ness in the right hand side of the *IAD* (and *LTI*) PDF is expected.

In summary, by introducing a *unified target separation* among all ATC, the variance in the approach phase can be proactively controlled. This may allow the possibility of having a closer separation spacing (by setting a smaller *TV* or smaller buffer Δ) and higher throughput with a maintained level of risk.

However, the other important side of the story is to provide controllers with a short-run feedback mechanism from the operation. For this purpose, we provide air traffic controllers with complementary values besides target value standards. Specifically, we also provide a Lower Bound (*LB*) for separation spacing and a corresponding probability γ, where it is desired that the proportion of paired landings with separation less than *LB* is less than γ – that is, $P\{IAD<LB\} <= \gamma$. Different values of *LB* can be specified for different probabilities γ. So, *LB* can be thought of as a function of γ, that is, $LB(\gamma)$.

3.2 Methodology to Set a Target Value and Lower Bound

To set suitable values for the aforementioned separation parameters, i.e. *TV* and *LB* (for a given probability), we need to observe the long-run system operations to get an insight about the steady state and dominant PDF of *IAD, LTI*, and the imposed control by ATC and pilots; call these long-run PDFs *process characteristics*.

Samples from the probability distribution of the imposed separation are not directly observable (Figure 4). A question is how to obtain this PDF using samples of other observable variables such as inter arrival time (IAT) to the terminal radar approach control (TRACON) area and *LTI* to the runway threshold. Xie et al. [6, 17] suggest that this transformation function can be modeled as an

M/N/1 queue, where the *IAT* is exponentially distributed and the imposed spacing distribution is approximately a normal distribution. They estimate the imposed normal distribution around the mode of the *LTI* PDF for the pool of all landings in both peak and non-peak periods. They do not separate peak and non-peak period operations.

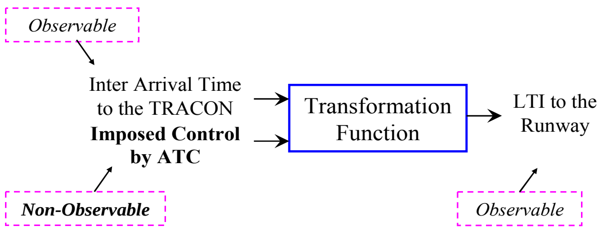

Figure 4. Observable and Non-Observable Distributions

We suggest that a more effective approach to address this problem is to find the distribution of *LTI* and *IAD* in peak periods. In this way, the main parts of the larger gaps (which are the result of large arrival gaps to the TRACON area) are systematically eliminated. Then the mode of this distribution represents approximately the mean (or the target value) of the imposed separation, which we assume is normally distributed as discussed earlier. We also assume that the standard deviation of the imposed separation can be estimated from the spread of the observed PDF – for example, by subtracting the observed mode and an observed low-probability quantile. The explicit formula for estimation of the standard deviation is given later in the algorithm, step 5.

Maintaining secure separations is not very challenging in a non-peak period. Problems generally occur in heavy traffic times when there is a tendency to land as many aircrafts as possible, and as a result, to reduce the separation. Therefore, we need to obtain the stable pattern of the system behavior, i.e. PDF of *IAD* (and *LTI*), in peak periods. In this research, we assume that periods of seven or more arrivals per quarter hour represent a peak period.

The target value should be set with respect to the following constraints

$$P\{LTI<ROT\} <= \alpha_R, \quad (9)$$

and

$$P\{\text{at least moderate wake vortex encounter}\} <= \alpha_{WV}, \quad (10)$$

where α_R and α_{WV} are very small values, e.g. 10^{-4}, representing acceptable runway and wake vortex related risk probabilities, respectively.

This paper does not address how to set the values for α_R and α_{WV} , but rather assumes the values are given. Also, this paper does not address calculation of the probability in (10). This could be accomplished through a variety of wake models such as the NASA AVOSS model [19] or the P2P model [20]. Here, we focus on computing the probability in (9).

Now, the goal of setting the target value *TV* is to shift the PDF of the landing time interval *LTI* so that the probability in (9) is satisfied (for simplicity, we ignore the wake vortex requirement (10) in this paper). As we have assumed, the target value *TV* is also the mode τ of the distribution of *LTI*. More specifically, *LTI* shall be shifted so that the average throughput is maximized subject to an upper limit of α_R in (9). If different values of the mode τ correspond to shifting the distribution of *LTI* up or down, then the problem can be stated as

$$\begin{aligned} &\text{maximize } E(\text{Throughput}) = \frac{1}{\mu_{LTI}} = \frac{1}{\tau + c} \\ &\text{subject to } P\{LTI(\tau) < ROT\} \le \alpha_R \\ &\qquad \tau > 0. \end{aligned} \quad (11)$$

where c is a positive constant equals to $\mu_{LTI} - \tau$.

$P\{LTI(\tau)<ROT\}$ gets larger as $TV_i=\tau$ decreases, i.e. $P\{LTI(\tau)<ROT\}$ is monotonically decreasing in τ (Figure 5). That is, a bigger overlap of *LTI* and *ROT* implies a bigger $P\{LTI(\tau)<ROT\}$. Point zero in this figure is the moment that the leading aircraft crosses runway threshold. Note that On the other hand, the average throughput is maximized for the lowest possible τ. Thus to solve this optimization model, it is enough to solve (9) in its equality form

$$g(\tau)= P\{LTI(\tau)< ROT\}-\alpha_R=0. \quad (12)$$

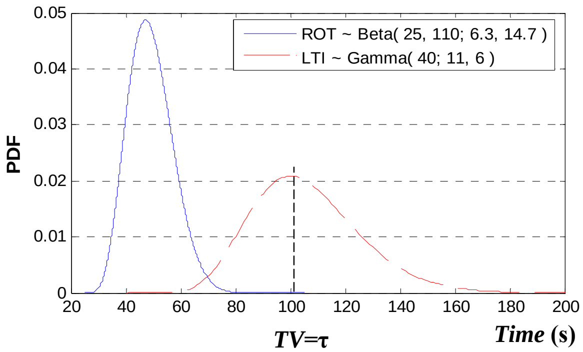

Figure 5. *TV* Location defines the overlap size of *LTI* and *ROT* [9]

We solve this equation for τ using the bisection search method since $g(\tau)$ is monotone in τ. $P\{LTI(\tau)<ROT\}$ should be calculated in every step of the bisection search algorithm for a given value of τ using stochastic simulation, for example.

The following algorithm completes and summarizes the discussion of setting the target separation spacing for a given pair of follow-lead aircraft in order to maximize throughput given an upper limit for $P\{LTI<ROT\}$.

The Algorithm

1. Decide on the arrival rate to represent peak landing periods. The mode of *LTI* PDF in these periods can represent the targeted value of the imposed separation in Figure 5. Collect a sufficient number of samples from (the stable pattern of) the process in peak periods over a long period of time during a given weather condition, e.g., instrument meteorological condition (IMC) in this paper.

2. Fit PDF's for *IAD* and *LTI* for different pairs of follow-lead aircraft. For example, [9] uses a gamma distribution fit. Obtain modes of these distributions in addition to other parameters. Also, fit a PDF. For example, [9] uses a beta distribution fit.

3. Based on outputs from step 2, and by using the bisection algorithm, solve (12) for τ for a given upper bound α_R for $P\{LTI<ROT\}$. This provides the time-based target value separation $TV_t^*=\tau^*$.

4. Estimate the average ground speed on the glide slope (the final path ending on the runway which has 3 degree slope in average as illustrated in Figure 2) by dividing the mode of the *IAD* PDF by the mode of the *LTI* PDF calculated in step 3. Multiply τ^* by the average speed to obtain the optimum nmi-based target value TV_n* separation.

5. Estimate the standard deviation σ of the overall imposed control by the group of ATC, which is assumed normally distributed. Knowing that $P(N(\mu,\sigma^2)<3\sigma)=0.0013$, estimate σ by $\hat{\sigma} = \left(\text{mode}\left(LTI\right) - F^{-1}\left[0.0013\right]\right)/3$ where F is the CDF of *LTI*. If the target values of ATC's were unified, this standard deviation would estimate the *stable system of chance-causes* deviations from the target value. Apply this step to *IAD* as well. In the long run, after completing the learning curve for the unified target-value separation, the standard deviation shall be considered as part of the separation standard so that any significant divergence from it shall be recognized.■

A Monitoring Mechanism

A monitoring mechanism is necessary to assure the process operates as it is designed and the risk is under control. One way is to count the number of times that *LTI* (or *IAD*) is below a certain threshold. If the fraction of observations below the threshold is too large, we conclude the process is out of control. For example, if $F(\cdot)$ is the CDF of the observed process, then it is expected that 100γ% of observations lie below $F^{-1}(\gamma)$. For $\gamma=0.02$, if more than 2% of IMC-peak period *LTI* falls under $F^{-1}(0.02)$, then we conclude that the runway related risk of $P\{LTI<ROT\}$ is out of control, and possible actions shall be taken to find any assignable causes, and make corrections as possible. This might be because the ATC targets smaller separation value, or because the variability has become larger or because of other assignable causes. A monitoring mechanism based on the 2-percentile allows judgment about the process in about 50 landings. On the other hand, if the monitoring mechanism is based on observing a simultaneous runway occupancy – which is a rarer event – it takes many more observations before a judgment can be made.■

3.3 A Numerical Example

Jeddi et al. [9] have studied the approach process at Detroit metropolitan Wayne county airport (DTW) using one week of multilateration track data. They provide PDF's for *IAD*, *LTI*, and

ROT under IMC after showing that samples from each one of these random variables are approximately independent. Then, assuming that these samples are from identical distributions, the "independent identically distributed" (i.i.d.) assumption, which is necessary for distribution-fitting purposes, is satisfied. They also demonstrate that based on their samples ROT_k and $LTI_{k,k+1}$ are approximately independent. To have sufficient data for the fitting purpose they aggregate the observations for all pairs of follow-lead aircraft that are subject to class-3nmi separation standard. In this section, we use their results to illustrate setting up statistically driven target values *TV* for in-trail threshold separation spacing.

Algorithm Illustration

1. We use an arrival rate threshold of seven or more per quarter-hour to represent peak periods. Assume that the one week observation is long enough to represent the steady state system behavior. These are also assumed in [9]. Reference [9] obtains 511 samples of *IAD* and 523 samples of *LTI* for the pairs subject to 3 nmi (or 2.5 nmi in special cases) minimum separation standard in Table 1.

2. According to [9], Gamma(1.5; 0.35, 6) is a reasonable PDF fit for *IAD* where 1.5 nmi is the minimum of *IAD* range, 0.35 is the scale parameter and 6 is the shape parameter (see Figure 6). The estimated distribution of *ROT* is Beta(6.1,15.4) in the range (25,110) s.

3. For this example, let α_R=0.001 and obtain the corresponding mode and shift of the *LTI* PDF to satisfy equation (12) using the bisection search algorithm and stochastic simulation. The expected value of the current P{$LTI(\tau_0)<ROT$} is calculated as 0.004, for τ_0=95 (or equivalently μ_0=106 for *LTI*) where 0 index indicates the current value. This is more than the desired level of 10^{-3}. Using the bisection algorithm, the optimal TV_t^* is calculated to be 101 s – that is, the PDF of *LTI* should be shifted by 6 seconds to the right. For $\alpha_R=10^{-4}$, TV_t^* shall be 110 s, i.e. a shift of 15 s is necessary.

4. From the information reported in [9], the average in-trail speed is 123 knots (=3600*3.25/95). Then the corresponding shift of *IAD* PDF is 0.2 (or 0.5) nmi to guarantee P{$LTI<ROT$} in 10^{-3} (or 10^{-4}) level, in the expected value sense, based on the samples in hand. In this case, nmi-based target value TV_n^* of separation for pairs of class-a shall be set to 3.45 (or 3.75) nmi.

5. F_{LTI} is the CDF of Gamma(40,11,6). Estimate the standard deviation of the normal control by

$$\hat{\sigma} = \left(\text{mode}(LTI) - F_{LTI}^{-1}[0.0013]\right)/3$$
$$= (95 - 13)/3 = 14\,\text{s.}$$

In the same manner, standard deviation of nmi-based imposed separation is 0.45 nmi.

Thus, we may consider the imposed control as N(3.25,0.45^2) (Figure 6). From (2), this result concludes that the imposed nmi-based separation $S=MS+\Delta+\varepsilon$ follows a N(3.25,0.45^2) distribution. In other words, if MS=3.0 nmi, then E(Δ) is 0.25 nmi and Var(Δ+ε) is 0.45^2 nmi^2. (Note that this is the amount of the variance that we considered for the variance reduction example earlier in section 3.) In the same manner, the left side of the *LTI* PDF is estimated by N(95,14^2) around the mode 95 s (Figure 7). Again, in the terminology of equation (2), the time-based separation S~N(95,14^2), i.e. E(MS+Δ)=95 s and Var(Δ+ε)= 14^2 s^2.■

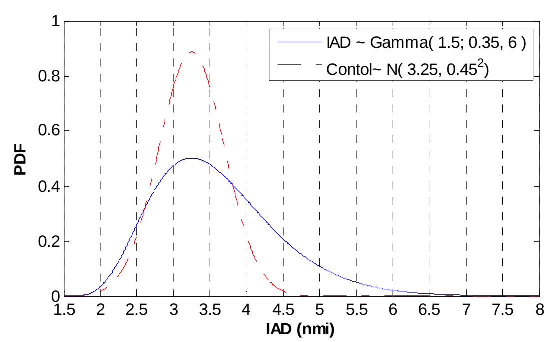

Figure 6. IMC-peak Period *IAD* and the Imposed Control

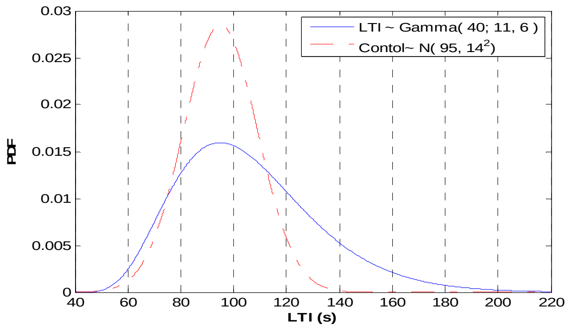

Figure 7. IMC-peak Period *LTI* and the Imposed Control

Monitoring Mechanism Illustration

F_{LTI} and F_{IAD} are the CDF of Gamma(40;11,6) and Gamma(1.5;0.35,6), respectively. $F^{-1}_{LTI}(0.02)\approx$ 63 s and $F^{-1}_{IAD}(0.02)\approx 2.2$ nmi. If among 100 IMC-peak period landings more than 2% of *LTI* (or *IAD*) occur under 63 s (or 2.2 nmi), conclude that the process does not operate as expected. Causes of such phenomenon shall be identified and appropriate changes in the process shall be applied.■

The current average throughput in IMC-peak periods is 8.5 arrivals per quarter-hour, which corresponds to a mean of 106 s for *LTI*. This provides runway related risk of 0.004 in peak-IMC periods. By the shift of +6 seconds, this risk is limited to 0.001, and the average throughput is 8.0 arrivals per quarter-hour. To secure the risk to 10^{-4}, it is necessary to reduce the average throughput to 7.4 arrivals per quarter hour (Figure 8a), i.e. to have +15 s shift to the right.

Figure 8a shows the relationship between average throughput and the runway related risk. Figure 8b represents the relation between safety (defined here to be 1 minus the runway related risk) and the average throughput for the obtained probability distributions of *LTI* and *ROT*. The risk is increasingly sacrificed as throughput grows. In other words, the second derivative of the risk (safety) in terms of the throughput is positive (negative). For example, raising the average throughput from 8.5 to 9 increases the risk by 0.006 (=0.01-0.004); however for the same amount of increase from 9.0 to 9.5, the risk increases by 0.015 (=0.025-0.01), which is 2.5 times 0.006. This demonstrates that the rate of the risk increase in throughput is more than linear.

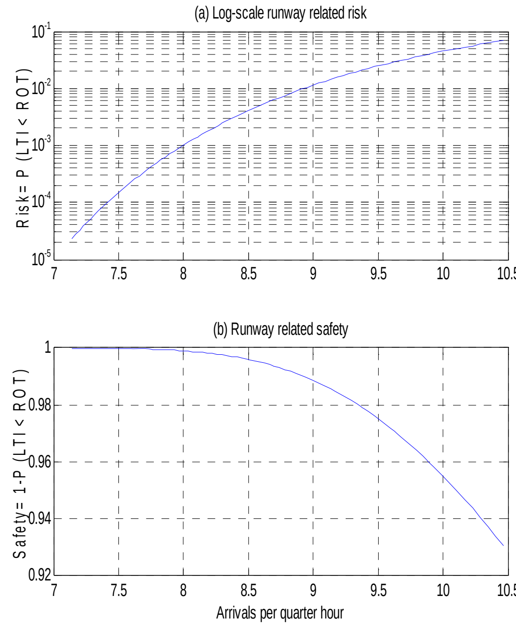

Figure 8. The Risk and Safety as Functions of Average Throughput

Fixing the buffer spacing among all ATC may decrease the variance of *LTI* . In such a case, the buffer spacing can be reduced without increasing the risk of *LTI*<*ROT*. Figure 9 shows P{*LTI*<*ROT*} as a function of reduction in *LTI* variance for a fixed amount of 8.5 arrivals per quarter-hour. The figure is drawn on a logarithmic scale. P{*LTI*<*ROT*} is calculated for *LTI*~Gamma(40;11·*e*,6/*e*), where *e* is a multiplier in [0.6,1.2]. The variance of the new distribution (after applying *TV*-based standards) is *e* times the original variance 27^2. From Figure 9, if the variance decreases to about 68% of the current value, runway related risk is reduced to 10^{-3} without changing the mean of *LTI* and/or average throughput.

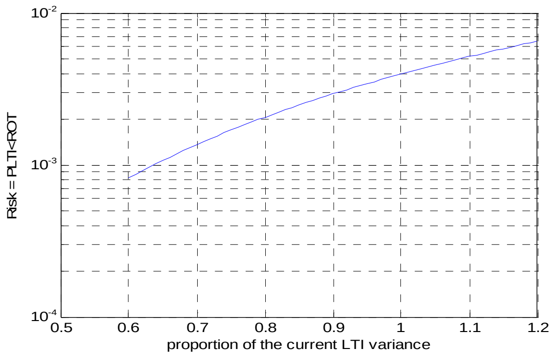

Figure 9. P{*LTI*<*ROT*} as a Function of Reduction in *LTI* Variance

4 Conclusion

In a process control approach, we modeled the influence of air traffic controllers on the aircraft separation as a random variable with a normal noise. The separation due to gaps in the schedule or arrival process is not the subject of this study and therefore is not included. We verified the observed variance in the current imposed control a result of two factors: 1) the natural inherent variability which we refer to as the stable pattern of chance causes, and 2) the variance outside the stable pattern of the chance causes referred to as assignable causes. The proposed separation model recognizes a systemic assignable cause in the current separation design. That is, for a given minimum separation standard, controllers add a buffer spacing and target a higher spacing than the allowed minimum separation standard in order to control the chance of simultaneous runway occupancy risk. The buffer spacing may differ among the controllers. We showed in (5) that this difference contributes to the overall variability of *IAD* (and *LTI*), and unifying buffer spacing (and target value) among controllers reduces the overall variance. We showed that if random separation errors of controllers have identical probability distributions, the variance reduction allows a closer spacing (in average) and higher throughput (and capacity utilization) with a maintained level of the risk.

One is not able to sample the imposed control. Therefore, the paper provided a statistical procedure to estimate the probability distribution of the imposed control given the PDF of *IAD* and *LTI* in peak periods. In this procedure, the mode of *IAD* estimates the average target value of all controllers. $\hat{\sigma} = \left(\text{mode}\left(IAD\right) - F^{-1}\left[0.0013\right]\right)/3$ estimates the standard deviation of the overall imposed nmi-based control where *F* is the CDF of *IAD*. We proposed a new approach to set separation standards using the statistical observations, so the term *statistical separation standards*. The new standard specifies not only a lower bound for the separation but also a standard for the *target value* and the *variance* of the process.

The standard target separation *TV* is calculated by shifting the PDF of *LTI* so that throughput is maximized and P{*LTI*<*ROT*} is remained under a very small given value. The standard variance is of the stable pattern of chance causes, which is calculated after target separation values are unified for all controllers and the learning curve is completed for the unified target value. We proposed a mechanism to monitor if the spacing process operates as desired over time. Results in [9] are used to illustrate the methodology, the monitoring mechanism, the possible variance reduction, and the increase of system utilization for a given level of the risk.

The paper focused on in-trail separation spacing under IFR considering only the runway related risk. However, the proposed model, methodology and procedures are readily adaptable for 1) en-route controlled separations, and 2) accounting for a very limited wake encounter risk. Obtaining further samples from the process, e.g., one month, is worthwhile for a better estimation of PDF of *IAD*, *LTI*, and *ROT*. More precise estimation of the imposed separation PDF is a valuable research venue. A more sophisticated monitoring mechanism of the separation process is necessary to explicitly provide in-action or medium-term feedbacks on the imposed target value and the process variance. Exploring these problems are subjects of our future research.

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Appendix I

Acronyms and symbols

CDF cumulative distribution function

DTW Detroit metropolitan Wayne county airport

IAD inter arrival distance

IFR instrument flight rule

IMC instrument meteorological condition

LTI landing time interval to the runway threshold

PDF probability distribution function

ROT runway occupancy time

SRO simultaneous runway occupancy

TRACON terminal radar approach control

s second

nmi nautical mile

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