ESTIMATING TAKEOFF THRUST FROM SURVEILLANCE TRACK DATA

Corresponding Author:
Lance Sherry
Center for Air Transportation Research at George Mason University, 4400 University Drive, MS:4A6, Fairfax, VA. 22030.
Tel:703-993-1711
Fax: 703-993-1521
Email: lsherry@gmu.edu

Saba Neyshabouri
Center for Air Transportation Research at George Mason University, 4400 University Drive, MS:4A6, Fairfax, VA. 22030.
ESTIMATING TAKEOFF THRUST FROM SURVEILLANCE TRACK DATA

Abstract:

Emissions inventory calculations for airports, using standard emissions inventory reporting methods, are inflated due an assumption of use of maximum takeoff thrust. To enhance the operational life of jet engines and reduce fuel burn, airlines frequently use reduced thrust (i.e. Derated or Flex Temperature thrust) for the takeoff procedure. Without access to internal airline “load sheet” paperwork or flight data recorders it is not possible to identify the thrust setting for each departure operation and account for the reduced thrust in emissions inventory calculations.

This paper describes an approach to estimate the takeoff thrust using a combination of radar surveillance track data, aerodynamic model, and weather data. Groundspeed and rate-of-climb from gear-up altitude to the thrust reduction altitude (1500’) are derived from surveillance track data. These values are combined with recorded wind speed data from airport weather data, an estimation of takeoff weight, and a steady-state aerodynamic model, to derive an estimate of takeoff thrust for each operation. Sensitivity analysis shows the importance of the estimate of takeoff weight in the calculations. An approach based on stage-length is used to estimate takeoff weight. A case study for flights from Chicago O’Hare airport exhibited an average takeoff thrust of 86% of maximum takeoff thrust. The implications and limitations of this approach are discussed.
1  INTRODUCTION

Air quality emissions inventories for airports are based on the modeling of the emissions from the aircraft engines during a landing and takeoff (LTO) cycle below the mixing height (typically 3000’ AGL). The inventory is computed for each phase of the LTO (e.g. taxi-out, takeoff, and initial climbout) by multiplication of time-in-phase, fuel burn rate, and an emissions index (see ICAO, 2011). The fuel burn rate and emissions indices for takeoff in the ICAO model are based on an assumption of maximum takeoff thrust.

Aircraft engine emissions, in particular oxides of nitrogen (NOx), however are highly sensitive to the engine thrust level and internal engine temperatures. First, higher thrust settings result in increased fuel flow rates that directly increase emissions. Second, for the case of NOx, higher thrust leading to higher engine temperatures results in an increase in the emission index. For other pollutants, Carbon Dioxide (CO2) and water vapor (H2O), the emissions index is constant as a function of thrust. For Carbon Monoxide (CO) or Unburnt Hydrocarbons (HC), the emissions index exhibits small increases as a function of thrust.

As a consequence, emissions inventory calculations using standard thrust levels and time-in-mode considerably over-estimate the emissions of these pollutants especially NOX (Hall, Mondoloni, & Thrasher, 2007; Herndon et al, 2008). To account for this phenomenon, improved estimates for takeoff thrust setting are required.

Actual thrust settings used for takeoff could be obtained from the airlines from “load sheets” or from Flight Data Recorders (FDR) or Flight Operational Quality Assurance FOQA) data. These data sets may require significant processing, may be subject to legal restrictions, and may be considered proprietary airline operational data. An alternate approach, described in this paper, is to use a combination of radar surveillance track data, weather data, and aerodynamic models to estimate the takeoff thrust.

Groundspeed and rate-of-climb from gear-up altitude (100’) to the thrust reduction altitude (1500’) are derived from surveillance track data. These values are combined with recorded wind speed data from airport weather data, and aerodynamic properties from a Base of Aircraft Data (BADA) for a steady-state aerodynamic model, to derive an estimate of takeoff thrust.

Sensitivity analysis shows the importance of takeoff weight (TOW) parameter in the accuracy of the thrust estimate. An approach based on stage-length is used to generate an estimate of TOW within +/-7% of actuals. A case study for track data from Chicago O’ Hare airport (ORD) shows that aircraft on average use 86% of maximum takeoff thrust. Limitations and implications of this approach are discussed.

This paper is organized as follows. Section 2 provides an overview of reduced thrust takeoffs. Section 3 describes the steady-state aerodynamic equations for takeoff and the required parameters. Section 4 describes derivation of all parameters for the estimation of thrust. Section 5 describes the results of a case-study for ORD. Section 6 includes a discussion of limitations and future work.
2 OVERVIEW OF REDUCED THRUST TAKEOFFS

Aircraft engines account for a large percentage of aircraft acquisition costs, while fuel and engine maintenance account for a large percentage of aircraft operational costs. It is not surprising therefore that airlines take significant steps to protect their investment and reduce their costs (Chenghong, 2002). One of the primary methods to reduced engine maintenance costs and to reduce fuel burn is through the use of reduced thrust for takeoffs.

The most common forms of reduced thrust are Assumed Temperature Thrust Reduction” (or “Flex Thrust”) and Derated Thrust. Flex Thrust is applied on a flight-by-flight basis when aircraft weight and aircraft performance allow a reduced thrust takeoff. Derated Thrust is a semi-permanent change to the engine’s maximum thrust (via changes to the engine management system) which is implemented when an airline expects that all operations performed will not require the full rated thrust of the engine.

In general, the thrust reduction available via Flex Thrust is limited to no more than 25% of the Rated Takeoff Thrust (FAA, 1988). Reduced takeoff thrust is not authorized for runways contaminated with standing water, slush or ice. Further, it is not authorized for use on wet runways unless a suitable stopping distance is available.

The thrust setting for takeoff is selected by the flightcrew using the Flight Management System (FMS) as shown in Figure 1 for the MD-11 (Honeywell, 1998). The Takeoff Thrust is shown in the line select field 1L (upper left). The 10% Derate thrust and 20% Derate thrust are selected in 2L and 2R. The Flex Temp thrust is selected in 1R. The Flex Temp entry by the flightcrew is an assumed temperature that is greater than the actual temperature. This results in a reduction of the maximum thrust that the engine can produce to avoid exceeding temperature limits in the core of

![FIGURE 1: Example Thrust Limit page for the MD-11 aircraft.](image)
the engine. Specifying the correct assumed temperature limits the engine thrust to the desired reduced takeoff thrust.

**Takeoff Profile**

A normal takeoff profile is summarized in Figure 2. Takeoff thrust is applied from the acceleration to rotate speed (VR) on the runway to the thrust reduction altitude (typically 1500’ AGL). Once the aircraft is airborne, an initial climbout segment from gear-up (typically 40’ – 100’ AGL) to the thrust reduction altitude (typically 1500’ AGL), is characterized by a constant airspeed ($V_2 + 10$ knots) and a fixed takeoff thrust setting. The rate-of-climb (i.e. flightpath angle) is a function of the aerodynamics of the aircraft given the airspeed and thrust setting.

**Constant Flightpath Angle = 10 to 15 degrees**

**Constant Speed = $V_2 + 10$ knots**

![Figure 2](image-url) Takeoff profile is defined by a constant airspeed ($V_{TAS} + 10$ knots) and constant takeoff thrust. The flightpath angle is a function of the aerodynamics of the aircraft and the airspeed and thrust.

### 3 STEADY-STATE EQUATIONS OF MOTION FOR TAKEOFF

The steady-state equations of motion for an aircraft can be characterized by Equation 1 (BADA, 2013 pg 14). Thrust ($T$) required to achieve a specific flight condition must overcome the: (1) Drag ($D$), (2) Weight ($m \cdot g$) component in the vertical longitudinal plane ($m \cdot g \cdot \sin(\gamma)$), and (3) inertia ($m \cdot dV/dt$).

\[ T = D + m \cdot g \cdot \sin(\gamma) + m \cdot dV/dt \]  

(1)

Drag is defined in Equation 2 based on atmospheric density ($\rho$), wing surface area ($S$), true airspeed ($V_{TAS}$) and the non-dimensional drag coefficient ($C_D$). Note, the Drag is a function of true airspeed and increases by the square of increase in true airspeed.

\[ D = \frac{1}{2} \cdot \rho \cdot S \cdot \frac{a \cdot V_{TAS}^2}{c} \cdot \frac{W}{S} \cdot C_D \]
\[ D = \frac{1}{2} \rho C_D S V_{TAS}^2 \]  
(2)

The drag coefficient can be estimated using the BADA coefficients CT1 and CT2 and the non-dimensional lift coefficient \( C_L \) as shown in Equation 3. The lift coefficient can be estimated as a function of weight (W), gravitational constant (g), atmospheric density (\( \rho \)), true airspeed (\( V_{TAS} \)), wing surface area (S) and flight path angle (\( \gamma \)) as shown in Equation 4.

\[ C_D = CT1 + (CT2 \cdot C_L^2) \]  
(3)

\[ C_L = \frac{(2W \cdot g)}{\left(\rho S V_{TAS}^2 \cos(\gamma)\right)} \]  
(4)

For the case of the constant speed climb, the equation for thrust can be reduced to:

\[ T = D + m \cdot g \cdot \sin(\gamma) \]  
(5)

**Data Sources**

For the purpose of this analysis some of the data can be derived directly from the surveillance data, other data has to be estimated or derived from secondary sources. The flightpath angle (\( \gamma \)) can be obtained directly from the surveillance track data. The true airspeed (\( V_T \)) is determined from the surveillance track data groundspeed (\( V_G \)), but needs the addition of headwind velocity (\( V_{HW} \)). \( V_{HW} \) is not available from surveillance data but can be obtained from airport wind data along with air density (\( \rho \)). The aerodynamic coefficients and wing surface area (S) are derived from the BADA. The takeoff weight (TOW) of the aircraft is not directly available from the data and must be estimated.

**Sensitivity Analysis**

A sensitivity analysis was conducted to assess the impact on thrust estimates based on the potential inaccuracy in TOW and \( V_{HW} \). The percentage of maximum takeoff thrust is computed for combinations of TOW and \( V_{HW} \). TOW ranges from Operating Empty Weight (EOW) to Maximum Takeoff Weight (MTOW). \( V_{HW} \) ranges from -5 knots headwind to +20 knots headwind. The example in Figure 3 is for an MD-83 aircraft using standard atmosphere with a 15 degree flightpath angle for takeoff.

The percentage of maximum takeoff thrust varies with headwind. The relationship between percentage of maximum takeoff thrust and headwind for different TOW’s is close to linear. A linear fit yields regression coefficients ranging from 0.94 to 0.97. The maximum change in percentage of maximum takeoff thrust due to a change in headwind from -5 knots to +20 knots is 4.9% for the MTOW scenario. The minimum change in percentage of maximum thrust due to a headwind change is 3.6% for the OEW scenario. The percentage of maximum takeoff thrust increases less than 0.01% for every knot of headwind.
The percentage of maximum takeoff thrust varies with TOW. The relationship between percentage of maximum takeoff thrust and TOW for different headwinds is linear. The maximum change in percentage of maximum takeoff thrust for TOW ranging from OEW to MTOW is 29.2% for the +20 knot headwind scenario. The minimum change in percentage of maximum thrust for TOW ranging from OEW to MTOW is 27.8% for the -5 knot headwind scenario. The percentage of maximum takeoff thrust increases on the order of 0.001 for every ton of TOW.

The percentage of maximum takeoff thrust is also affected by the flightpath angle. The percentage of maximum takeoff thrust is linear to changes in flightpath angle. The percentage of maximum takeoff thrust changes 0.05% for every 1 degree change in flight path angle.

In summary, within the overall fidelity of the model, the TOW has the most significant effect on the estimation of takeoff thrust.

4 ESTIMATING TAKEOFF THRUST FROM SURVEILLANCE TRACK DATA

The model for estimating takeoff thrust from surveillance track data, airport surface weather data, and aerodynamic model data is shown in Figure 4.
FIGURE 4 Schematic of the model used to estimate takeoff thrust. Data sources include surveillance track data, aerodynamic properties from the BADA, atmospheric data, and an estimate of takeoff weight.

Aerodynamic Model

The equations for CL, CD, Drag and Thrust are listed above (Equations 2-5). The coefficients for each aircraft type are obtained by look-up in the BADA (EuroControl, 2012). The aircraft type is contained in the surveillance track data.

The estimated thrust for takeoff is divided by the number of engines to calculate the estimated thrust for each engine ($T_{Eng}$). The number of engines (n) is determined by look-up in the BADA based on the aircraft type in the surveillance track data.

$$T_{Eng} = \frac{T}{n}$$ (6)

The maximum takeoff thrust for each engine $T_{Max}$ is estimated from the equations from BADA for each aircraft type. The coefficients in the equation are looked-up in the BADA based on the aircraft type. The pressure altitude ($H_p$) is set to 700’ + airport elevation. Seven hundred feet is halfway between 1500’ and 100’ AGL. The equations for jet engines is shown in Equation 7/

$$T_{Max} = TC1 \cdot (1-(H_p/TC2)) + (TC3 \cdot H_p^2)$$ (7)

The percentage of maximum takeoff thrust (per engine) is $T_{Eng}/T_{Max}$

Aircraft Trajectory
The flightpath angle is best derived from the horizontal distance traversed (d) from 40 ft AGL to 1500’ AGL (h). Due to the noise inherent in surveillance track data altitude, the average of a cluster of altitudes around 40’AGL is used to establish the start altitude and distance, and around 1500’ for the end altitude and distance.

\[ \gamma = \tan^{-1} \left( \frac{d}{h} \right) \]  

(8)

True airspeed \( (V_{TAS} \text{ - m/s}) \) is computed at each time step from Groundspeed \( (V_G \text{ - m/s}) \), aircraft heading with respect to north \( (\theta \text{ - radians}) \) and headwind \( (V_{HW} \text{ - m/s}) \) derived from Windspeed \( (V_W \text{ - m/s}) \) and Wind direction with respect to north \( (\phi \text{ - radians}) \). Wind data for the time of the flight operation can be obtained from the Aviation System Performance Metric (ASPM) data base (FAA, 2013) or the Rapid Update Cycle (RUC) database (NOAA, 2013). The equation for \( V_G \) is shown in Equation 9. The equation for \( V_{TAS} \) is shown in Equation 10.

\[ V_G = (d_i - d_{i-1})/\Delta t \]  

(9)

\[
V_{TAS} = \frac{[(V_G \sin\theta) \quad (V_W \sin\phi)]}{[(V_G \cos\theta) \quad (V_W \cos\phi)]}
\]

(10)

**Estimated Takeoff Weight (TOW)**

The estimate for TOW is calculated by the summation of: (i) OEW for the aircraft, (ii) stage-length fuel required \( (F_{SL}) \), (iii) 45 minutes reserve fuel \( (F_{RSRV}) \), (iv) a 60 minute alternate fuel \( (F_{ALT}) \), and the passenger and cargo payload \( (P_W) \). The OEW is looked-up in the BADA for the aircraft type (Equation 11).

\[ TOW = OEW + F_{SL} + F_{RSRV} + F_{ALT} + P_W \]  

(11)

\[ F_{SL} = [0.5 \text{ Hour} \bullet FBR_{CLB}] + [(SL/V_{Czr}) \bullet FBR_{Czr}] \]  

(12)

\[ F_{RSRV} = 0.75 \text{ Hour} FBR_{Czr} \]  

(13)

\[ F_{ALT} = 0.5 \text{ Hour} \bullet FBR_{Czr} \]  

(14)

\[ P_W = 0.8 \bullet \text{Seats} \bullet 95.25\text{kg/pax} \]  

(15)

The stage-length fuel required \( (F_{SL}) \) has two components. The climb fuel assumes a 30 minute climb with a climb fuel burn rate \( (FBR_{CLB}) \). The cruise fuel is calculated based on the stage-length \( (SL) \), cruise velocity \( (V_{Czr}) \), and cruise fuel burn rate \( (FBR_{Czr}) \) at FL350. See Equation 12. The cruise velocity and cruise fuel burn rate are looked-up from the BADA for the aircraft type.
Fuel reserve (F_{RSRV}) and alternate fuel (F_{ALT}) are based on 45 minutes and a 30 minute flight using the cruise fuel burn rate.

Payload weight (P_W) is based on the assumption of an 80% load factor. The number of seats are obtained by look-up in the BADA. The average weight per passenger is assessed at 95.52 kg.

For a sample of 1000 B737-class aircraft, the algorithm described above yielded a range of error from -7% to +19%. The distribution of error exhibited on mode with a right-tail. The average error was 2% and a standard deviation of 4%. A comparison of actual and estimated TOW for these flights shows a model tendency for over-estimation for (relatively) low TOW and under-estimation for high TOW (Figure 5). Differences between the estimate and actual may be the result cruise speeds, fuel tinkering, and variations in the load factor.

![Figure 5](image)

**FIGURE 5** Estimated TOW vs Actual TOW for one thousand 737-class U.S. domestic flights. Estimated TOW is over-estimated on the low and tends to be under-estimated on the high-end

5 **CASE-STUDY: ORD TAKEOFFS**

Thrust for takeoff was estimated based on surveillance track data for 250 flights departing Chicago O’Hare airport on 21 January 2011. The distribution of flights by estimated takeoff thrust as a percentage of maximum takeoff thrust exhibits two modes: a mode centered around 75% and a mode at 100% (see Figure 6). On average, the flights exhibited an average takeoff thrust 86% of the maximum takeoff thrust. The standard deviation was 11%.
6 CONCLUSIONS

The model described in this paper provides a means to estimate takeoff thrust for emissions inventory calculations that takes into account the use of reduced thrust (i.e. Flex Temperature or Derate) settings. The method combines surveillance track data, with airport weather data and an aerodynamic model. The model is most sensitive to the estimate for takeoff weight up to 30%. Headwind contributes up to 5% to the accuracy of the estimate. Flightpath angle contributes less than 1% for every degree of error.

An approach to modeling takeoff weight was developed and validated with proprietary airline data. For the sample data set, the estimate of TOW exhibited an average error of 2%, with a range of -7% to +19%. The estimates used in the analysis described in this paper did not use winds aloft data to estimate of cruise speeds. This could be added to the model.

The model described in this paper assumes all flights use the reduced thrust setting. The model could be improved by adding logic to account for local runway or weather conditions that would prohibit use of reduced thrust.

With the limitations described above the results of the model described in this paper could be used to define the lower end of the range of takeoff thrust with the upper end of the range defined by the assumption of maximum takeoff thrust used by the standard emission inventory reporting methods.

Acknowledgements

Akshay Belle for several technical contributions. Gareth Horton, Hazel Pearce (Ricardo-AEA), Damon Fordham (Cadmus Group), Terry Thompson, Stephen Augustine (Metron Aviation),
Paula Lewis (FAA), John Shortle (Center for Air Transportation Research at George Mason University). This research was funded by ACRP 02-41 and by internal CATSR research foundation funds.

**References**


