Estimation of Potential Conflict Rates as a function of Sector Loading

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Abstract— **Automated separation assurance systems are being developed to reduce controller workload and increase airspace capacity of the National Airspace System (NAS). To evaluate these systems, a measure for conflict risk is required. The objective of this analysis is to estimate the rate at which flights enter a course of potential conflict or collision under different traffic loads. Conflict rates are estimated *under the assumption of no conflict resolution*. In other words, the analysis is aimed at estimating precursors to actual conflicts. The conflict rates are estimated (a) using a NAS-wide simulation, (b) for a futuristic NAS-wide 1.5X traffic schedule, (c) for airway routes and great circle routes, (d) for different conflict-volume dimensions (conflict types), and (e) for different sector traffic loads. Conflict types include loss of separation (LOS), critical loss of separation (CLOS), near mid air collision (NMAC), and mid air collision (MAC). The simulation of flight trajectories and detection of conflicts are done using Future Air Traffic Management Concepts Evaluation Tool (FACET). A hybrid analytical-simulation approach is also used to estimate the rate of MACs. A key result is that the rates of NMACs and MACs for the airway routes are higher than the corresponding rates for the great-circle routes. The results also show that conflict rates follow the quadratic relationship with respect to flight count.**

Index Terms— **Conflict rate, separation assurance, near mid air collision**

I. INTRODUCTION

U.S. air traffic is expected to grow at a rate of about three percent per year [1]. At this rate, traffic will have grown by about thirty percent by 2020. To enable future growth, two major concepts are being developed to perform monitoring and automated separation assurance. One is a distributed system called Autonomous Flight Management (AFM), where each aircraft is equipped with its own conflict detection and resolution (CD&R) capabilities [2][3]. The other is a centralized ground-based system called the Advanced Airspace Concept (AAC), where the conflict detection and resolution is provided to the aircraft by a ground-based automation [4][5][6]. Both concepts have the potential to reduce controller workload by automatically separating some aircraft.

Before such systems can be implemented, they must be shown to be safe. For example, [7] provides a safety analysis of the AAC concept using fault trees and event trees. The initiating event of the tree in that paper is the occurrence of two aircraft being on course for a NMAC. The objective of this analysis is to estimate the rates of these initiating events to support evaluation of candidate separation assurance systems.

More specifically, the objective of this analysis is to estimate conflict rates under high traffic loads. Conflict rates are estimated *under the assumption of no conflict resolution*. In other words, the objective is to estimate the rates at which aircraft are *on course* for a conflict – which might be considered a precursor or an initiating event to an actual conflict. We consider four types of conflicts, defined with respect to their volumes: LOS, CLOS, NMAC and MAC. These associated volumes are defined in Section II.

We estimate conflict rates for high altitude sectors in the ZAU center using a NAS simulator under a 1.5X traffic scenario (1.5 times the present day traffic). Two sets of 1.5X traffic are considered. In the first set, flights are assigned airway routes (AR). In the second set, flights are assigned great-circle routes (GCR). The simulation of flight trajectories and detection of conflicts are done using the Future Air Traffic Management Concepts Evaluation Tool (FACET) [8]. Multiple simulation runs are conducted by stochastically varying the scheduled departure time for each flight. For great-circle routes, the MAC rate is also estimated using a hybrid analytical-simulation model.

The output of the simulation analysis is a series of plots showing the conflict rate as a function of flight count. A key result is that the rates of NMACs and MACs for the airway routes are higher than the corresponding rates for the great-circle routes. This indicates that great-circle routes are more spread out and have fewer intersections than the airway routes. Thus, if aircraft fly their user-preferred shortest distance, the resulting rate at which aircraft are on course for an NMAC or MAC may decrease. The results also show that the conflict rate follows a quadratic relationship with respect to flight count, as might be expected from models in the literature (see discussion in Section II).

II. Background

A. Related Research

One approach to estimate conflict rates is to use an analytical model. For example, Geisinger [9] developed a three dimension analytical model to compute the rate of conflicts at an intersection of flight paths. Geisinger considered intersecting paths and parallel paths. For the intersecting path scenario, eight different cases were considered based on the relative position of the two aircraft. For the parallel-path scenario, three cases were considered based on relative direction (opposite, or same) of the aircraft.

Other analytical models differ in terms of the conflict geometry, flight paths, and the flow rate assumed. For example, May [10] developed a mathematical model to estimate the potential NMAC in a volume of airspace about which traffic patterns are known. The model computes the expected number of NMAC per year for a given airspace. Siddiqee [11] developed a mathematical model to predict the expected number of potential conflict situations at the intersection of jet routes. Given the intersection angle of two routes, the average flows and speed of aircraft, the model predicts the average rate of potential conflicts. Dunlay [12] developed two mathematical models, one for crossing conflicts and one for overtaking conflicts. Hu et al. [13] developed a model to compute the probability of conflict by modeling aircraft motion with a scaled Brownian motion perturbation. Barnett [14] developed a stochastic mathematical model for collision risk assessment of a free-flight concept.

One problem with analytical models is that they do not take into account complicating factors such as the specific route structure that may exist in the airspace. Related simulation models include the following: Willemain [15] developed a simulation model to assess the impact of factors such as sector entry time, sector flight count, orientation of flight path, and distribution of airspeeds on free-flight risk measures. Kochenderfer et al. [16] estimated the probability of a MAC given an NMAC using surveillance data, an encounter model, and a three dimensional aircraft wireframe model. Jardin [17] showed that under free routing conditions, the expected number of conflicts is well represented by a binomial random variable model. Also the instantaneous probability of conflict, i.e., the probability of flight i conflicting with any other aircraft j at a given instance in time, is $9*10^{-6}$ for airway routes and $7*10^{-6}$ for great circle routes. The expected number of conflicts per flight in class A airspace with 3000 active flight was estimated to be 0.027 for airway routes and 0.021 for great circle routes.

The main contributions of this analysis are that we estimate conflict rates (a) from a NAS-wide simulation (using NASA's FACET), (b) for a futuristic NAS-wide 1.5X traffic schedule, (c) for airway routes and great circle routes, (d) for different conflict-volume dimensions, and (e) for different sector traffic loads. While existing studies may be suitable to address some of these requirements, the approach adopted in this study was chosen to satisfy all requirements.

B. Conflict Definition and Rate of Conflict

With an increase in traffic, the resulting increase in the expected number of conflicts is expected to follow a quadratic model [18]. To intuitively understand this, consider the intersection of two flight routes, as shown in Fig. 1. Suppose that the traffic along *one* of these routes is increased by 20%. Then it is expected that the number of conflicts at the intersection will increase by 20%. Now suppose that the traffic along the other route is also increased by 20%. By applying the same logic the factor by which the conflicts will increase at the intersection is 1.2*1.2, i.e., 44%. This is true for two routes intersecting at any angle. For instance when the angle between the routes is zero, the flights are flying along the same route in the same direction, and a potential conflict is due to passing. When the angle between the routes is 180°, the flights are flying along the same route in the opposite direction, and the conflict is due to a head-on approach.
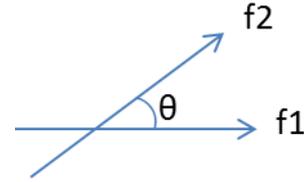
Fig. 1.   Intersecting Routes

In this analysis, a conflict is defined as an instance when two flights vectors are in proximity closer than the specified separation minima. Instances where three or more flights are simultaneously in conflict are considered two at a time. For instance, if three flight vectors A,B,C are in conflict, then the conflicts are considered as AB, BC and AC.

Table 1 shows the lateral and vertical separation minima for conflicts assumed in this analysis.

TABLE 1
SEPARATION MINIMA FOR CONFLICT TYPES

Conflict Type	Lateral	Vertical
LOS	5NM	1000 ft
CLOS	1.1NM	100ft
NMAC	500ft	100ft
MAC	100ft	30ft

The LOS lateral and vertical separation minima are the en-route separation minima as specified by the Federal Aviation Administration (FAA). The CLOS horizontal minimum is the approximate distance equivalent of five seconds before collision between flights traveling straight at each other at 400 knots each (i.e., 800 knots relative velocity). The NMAC minima are as defined by FAA and Eurocontrol [19][20]. The MAC minima correspond roughly to the size of an aircraft.

III. Methodology

A simulation-based model is used to estimate the rate of LOS, CLOS, NMAC and MAC. Because of rare-event

limitations in estimating collisions (MACs), a hybrid simulation-analytical approach is also used, and both approaches are compared.

A. Simulation Model

Fig. **2**. shows the simulation-based methodology. Two sets of 1.5X traffic are considered. In the first set, flights are assigned airway routes. The airway routes are generated from flight plans obtained from the real Traffic Flow Management System (TFMS) data. In the second set, flights are assigned great-circle routes from origin to destination. The simulation of flight trajectories and detection of conflicts is done using FACET. Fifty simulations are run for each set of traffic, where stochastic variability between the runs is given by varying the scheduled departure time for each flight. This is chosen according to a uniform distribution with a minimum value of zero and a maximum value of thirty minute. The simulations are run using a five second time step.

The stochastic variability between the run is provided to effect the time and location of conflicts. The runs can be randomized further by varying the speed, altitude, and routes (in case of airway routes). It is expected that a purely randomized traffic scenario would exhibit a strong quadratic relationship between number of conflicts and number of aircrafts flying [14][21].
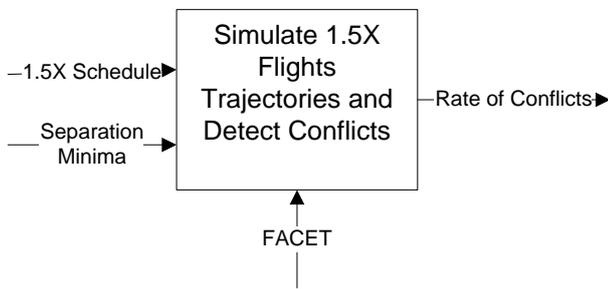
Fig. 2. Methodology

FACET is a fast time simulator. It can (among other capabilities) simulate 4D flight trajectories and report instances of conflict at every time step. The information required by FACET to simulate flight trajectories are the flight schedules (flight plan and departure time) and flight type. To detect conflicts, a separation minima is specified. For each run, FACET outputs a conflict file and a flight location file. The conflict file contains, for every time step, a list of flight pairs having separation less than the specified minimum separation and their coordinates (latitude, longitude and altitude). The exact lateral distance between flights in conflict is computed using the Haversine distance formula [22]. The conflicts are further classified as LOS, CLOS, NMAC or MAC based on Table 1. The flight location file contains the sector and flight-level information for each flight at every time step. The conflict file and flight location files are processed to determine the flight count and conflict count in each fifteen minute window in each sector.

1) Scope of Simulation

The scope of the simulation is limited by the simulation run time. The two factors that influence the simulation run time in FACET are (1) the area of airspace for which conflict detection is performed, and (2) the resolution at which the conflict detection is performed.

FACET can perform conflict detection for entire National Airspace System (NAS) or for individual centers. In this analysis, conflict detection is restricted to high altitude sectors in the Chicago center. These sectors were selected because of their high complexity. Six out of the ten high altitude sectors in Chicago center are among the top fifty complex sectors in the NAS, based on the dynamic density metric [23].

FACET simulations can be run with a user-specified time step. A smaller time step gives better resolution, but requires more simulation time. A larger time step is faster, but may miss some conflicts. In this analysis, a five-second time step is used. This provides a balance between simulation time, in which multiple replications can be performed in a reasonable time, and simulation resolution for estimating conflicts (see discussion in next section). With these settings it takes approximately three hours per simulation run, for high altitude ZAU sectors.

2) Conflict Detection

The degree to which conflicts are detected depends on the resolution of the 4D trajectories. At a resolution of 5 seconds, FACET cannot detect all instances of conflict for each conflict type, since two aircraft may enter and leave the conflict region between successive time steps. This is particularly true for collisions, where the conflict region is small relative to the distances traveled by aircraft during one time step.

We estimate the probability that FACET detects a conflict using the following equation:

$$\Pr\{Detection\} = \frac{T_C}{T_R} \qquad (1)$$

where T_C is the traverse time across the conflict area, and T_R is the time step of the simulation (e.g., five seconds). Using a coordinate system relative to the position of aircraft 1 (AC1), the traverse time of aircraft 2 (AC2) across the conflict area is the length of the traverse chord divided by the relative velocity between the two aircraft (Fig. **3**).
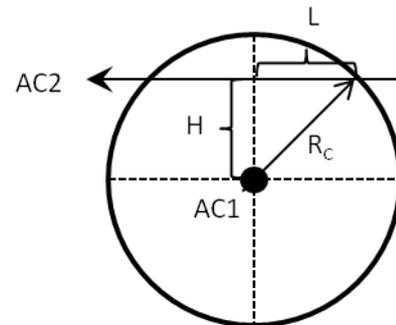
Fig. 3. Traverse Across Conflict Area

Then T_C is estimated using:

$$T_C = \frac{2L}{V_R} \qquad (2)$$

Where, L is half the traverse distance given by $\sqrt{R_C^2 - H^2}$, R_c is the lateral separation minima of the conflict type from Table 1, H is the perpendicular distance from center of circle to the traverse chord. V_R is the relative velocity of the two aircraft, given by $|V_{AC1} - V_{AC2}cos\theta|$, where V_{AC1}, V_{AC2} are the velocities of the aircrafts, and θ is the angle between the aircraft.

Table 2 shows the resulting probability of FACET detecting a conflict, obtained via Monte-Carlo simulation. In the simulation, H is assumed to be uniformly distributed on $[0, R_c]$, V_{AC1}, V_{AC2} are assumed to be uniformly distributed between 400 and 450 knots, and θ is assumed to be uniformly distributed between 0 and 360 degrees. The vertical dimension is ignored in this analysis. With a five-second time step, FACET detects nearly all instances of LOS and CLOS. There is a forty percent chance that FACET detects a NMAC and a fifteen percent chance that it detects a MAC.

TABLE 2
PROBABILITY OF CONFLICT DETECTION BY FACET

Conflict Type	Pr{Detection}
LOS (5NM)	0.999
CLOS (1.1NM)	0.981
NMAC (500ft)	0.395
MAC (100ft)	0.148

To improve the count of NMAC and MAC, we apply a closest-point-of-approach (CPA) algorithm [24]. First, trajectories of all conflict pairs that result in a CLOS are identified. FACET can detect most instances of CLOS using a 5-second time step (as estimated by the analysis in Table 2). A continuous trajectory is generated by linearly interpolating between successive points given at 5-second intervals. Assuming straight-line trajectories, simple geometric arguments yield the closest point of approach over a 5-second interval [24]. If the CPA is less than the NMAC minima, the count of NMAC is incremented. Similarly, if the CPA is less than the MAC minima, the count of MAC is incremented.

Since airway routes are not always straight. The CPA algorithm is applied only to the GCR simulation's conflict output. By doing so, most instances of NMAC and MAC are detected for the GCR case.

B. Analytical Model

We also estimated the rate of MAC using a simple geometric argument and the following equation:

$$\lambda_{MAC} = Pr\{MAC|NMAC\} * \lambda_{NMAC} \qquad (3)$$

where λ_{MAC} is the expected number of MACs per flight per fifteen minutes in a sector, λ_{NMAC} is the expected number of NMACs per flight per fifteen minutes in a sector, and $Pr\{MAC|NMAC\}$ denotes the conditional probability of a MAC given that an NMAC has already occurred. To compute λ_{MAC}, we estimate λ_{NMAC} from simulation and then compute $Pr\{MAC|NMAC\}$ analytically.

One way to estimate the probability of a MAC given an NMAC is to divide the MAC area by the NMAC area, as illustrated in **Fig 4**. This assumes a uniform distribution of flight trajectories throughout space and ignores the vertical dimension. This also assumes that aircraft are treated as point masses. With this argument, the probability of a MAC given an NMAC is [7]:

$$Pr\{MAC|NMAC\} = \left(\frac{Lat_{MAC}}{Lat_{NMAC}}\right)^2 \qquad (4)$$

where Lat_{MAC} and Lat_{NMAC} are the lateral separation minima for MAC and NMAC shown in Table 1. Reference [7] also takes into account the horizontal cross section of the aircraft, treating the aircraft as a circle from a top-down view.

Equation (6) gives the probability of a MAC given an NMAC in two dimensions only. In three dimensions, a similar argument gives:

$$Pr\{MAC|NMAC\} = \left(\frac{Lat_{MAC}}{Lat_{NMAC}}\right)^2 * \left(\frac{V_{MAC}}{V_{NMAC}}\right) \qquad (5)$$

where V_{MAC} and V_{NMAC} are the MAC and NMAC vertical separation minima shown in Table 1. By plugging in the values of, Lat_{MAC}, Lat_{NMAC}, V_{MAC}, and V_{NMAC} from Table 1 into (6) and (7), $Pr\{MAC|NMAC\}$ is estimated to be 0.04 for the 2D case and 0.012 for the 3D case.
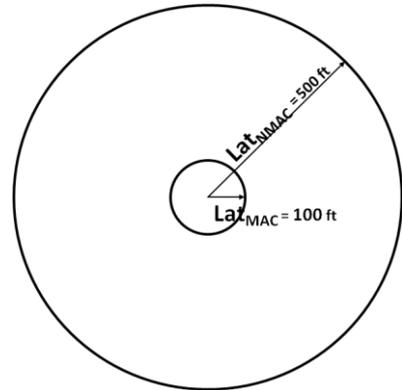
Fig 4. MAC given NMAC

The probability of a MAC given an NMAC is also estimated to be 0.1 in EUROCONTROL's Aircraft Collision Avoidance System (ACAS) program [20]. This is based on the estimated rate at which NMAC and MAC occur in European airspace. In [20] the rate of NMAC is estimated to be $3 * 10^{-7}$ per flight hour and the rate of MAC is estimated to be $3 * 10^{-8}$ per flight hour. By dividing the rate of MAC by the rate of NMAC, the probability of MAC given NMAC is estimated to be 0.1. Another estimate of this probability is given in [16]. The authors used an encounter model, a three dimensional aircraft wireframe model, and surveillance data to estimate the probability of a MAC given an NMAC as 0.01 [16]. Table 3 summarizes the estimates of $Pr\{MAC|NMAC\}$ discussed here.

TABLE 3
PROBABITLITY OF MAC GIVEN NMAC

Reference	Pr{MAC|NMAC}
[20]	0.1
[16]	0.01
Eq. (4) (2D)	0.04
Eq. (5) (3D)	0.012

IV. RESULTS

A. Conflict Rate per Sector

We simulate one day of traffic using FACET and repeat fifty times, varying the scheduled departure time for each flight. For each run, the flight count in each sector (super high sectors in ZAU) and the corresponding conflict count are computed for every fifteen-minute time interval. For a given flight pair, only the first instance of a conflict is taken into account. Otherwise, because conflicts are recorded every five seconds, a conflict may be reported in more than one time window. The flight counts in each fifteen-minute interval from all fifty runs are then binned in increments of five (0-5, 6-10, …) and an average of conflicts corresponding to each flight-count bin is computed.

Fig 5 to Fig 8 show the expected conflict count (LOS, CLOS, NMAC, and MAC) per fifteen minutes in a sector as a function of flight count for all ultra high altitude sectors in ZAU. The results come from FACET simulation output and do not involve any of the analytical extensions discussed in the previous section. As a frame of reference, current monitor-alert-parameter (MAP) values are around 20, so flight counts of 40 on the x-axis correspond to roughly twice that of current sector capacities.

A quadratic curve fits well in each case, with an R^2 of .98 or better, except for the GCR MAC counts. The quadratic model is expected and consistent with discussion in the literature (e.g., [14]). Section II-B gave an intuitive explanation for the quadratic model.

Comparing these figures, the main conclusion is that airway routes result in much higher conflict rates compared to great-circle routes, when smaller conflict regions are considered (e.g., for NMAC and MAC). This is due to the structured nature of trajectories along airway routes. The forced intersection points along the routes lead to higher probabilities of collisions, compared with less-structured great-circle routes. However, this difference diminishes when larger conflict regions are considered. For example, there is a difference of 2-45% (depending on sector traffic) in LOS rates for airway routes and great-circle routes. This difference is much larger in case of NMAC and MAC rates, which are in order of 400-900%. This result is consistent with findings in [21], where the difference in total LOS counts for airway routes and great-circle routes is reported to be 13%.
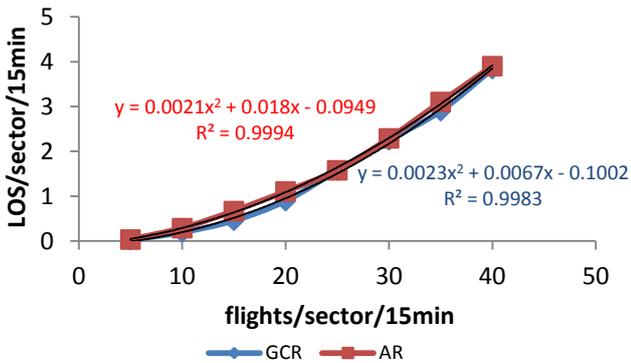
Fig 6. CLOS per sector per fifteen minute, for super high ZAU sectors
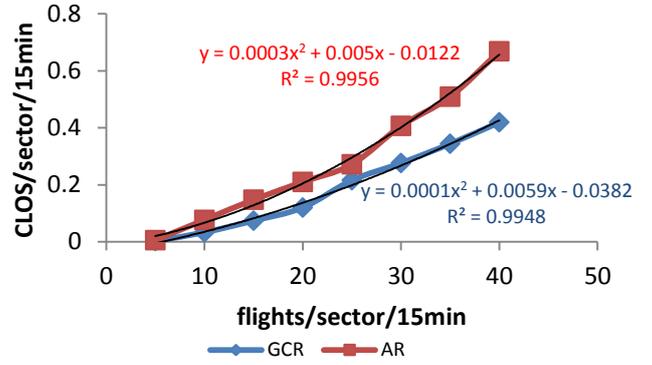
For the CLOS chart:
$$y = 0.0003x^2 + 0.005x - 0.0122$$
$$R^2 = 0.9956$$
$$y = 0.0001x^2 + 0.0059x - 0.0382$$
$$R^2 = 0.9948$$

Fig 7. NMAC per sector per fifteen minute, for super high ZAU sectors

For the NMAC chart:
$$y = 5E\text{-}05x^2 + 0.0003x + 8E\text{-}05$$
$$R^2 = 0.9857$$
$$y = 1E\text{-}05x^2 - 0.0003x + 0.0015$$
$$R^2 = 0.9806$$

Fig 8. MAC per sector per fifteen minute, for super high ZAU sectors

For the MAC chart:
$$y = 4E\text{-}05x^2 - 0.0005x + 0.0034$$
$$R^2 = 0.981$$
$$y = 1E\text{-}06x^2 - 1E\text{-}05x + 4E\text{-}05$$
$$R^2 = 0.6769$$
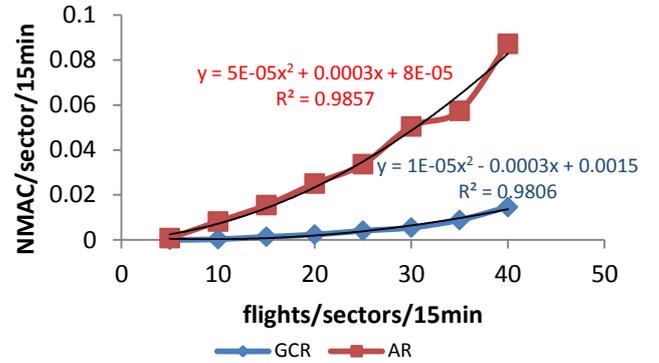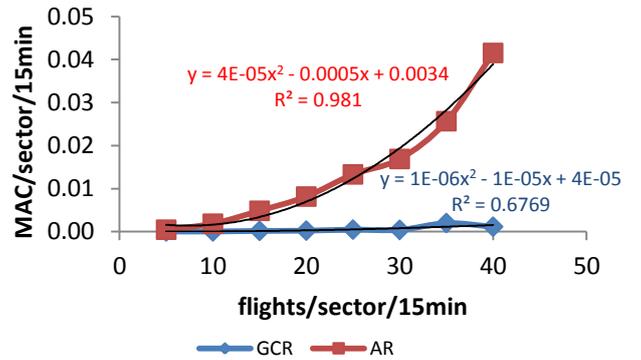
The above analysis is also performed for individual sectors. Fig. 9 shows for GCR, the expected LOS count as function of flight count for each sector in ZAU center. As depicted, for the same flight count, the rates of conflict differ from sector to sector, indicating different route structure within each sector.

Fig 5. LOS per sector per fifteen minute, for super high ZAU sectors

For the LOS chart:
$$y = 0.0021x^2 + 0.018x - 0.0949$$
$$R^2 = 0.9994$$
$$y = 0.0023x^2 + 0.0067x - 0.1002$$
$$R^2 = 0.9983$$
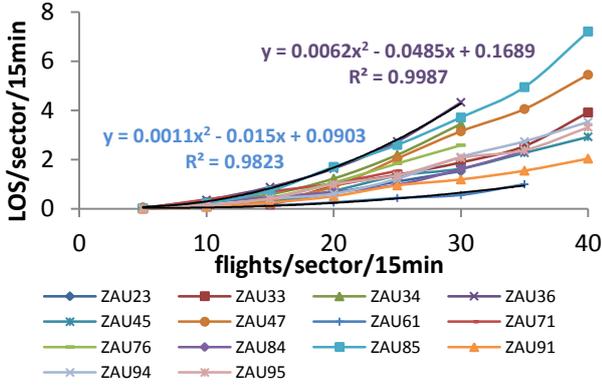
Fig. 9 LOS per sector per fifteen minute, for GCR routes, by sector.

B. Conflict Rate per Flight per Sector

In this section, we consider the conflict rate *per flight*. The expected number of conflicts per flight per fifteen minutes in a sector is derived by dividing the expected total number of conflicts per fifteen minutes in a sector by the respective flight count bin. These are shown as a function of flight count in Fig 10 to Fig 13. The relationship in this case is approximately linear (as expected).
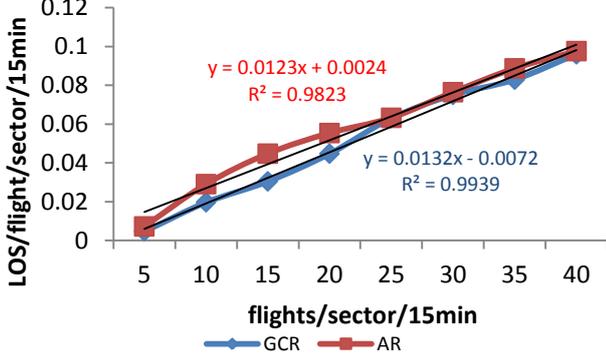
Fig 10. LOS per flight per sector per fifteen minute, ZAU super high sectors
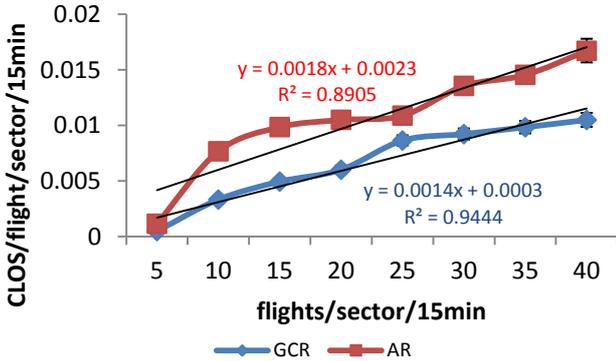
Fig 11. CLOS per flight per sector per fifteen minute, ZAU super high sectors
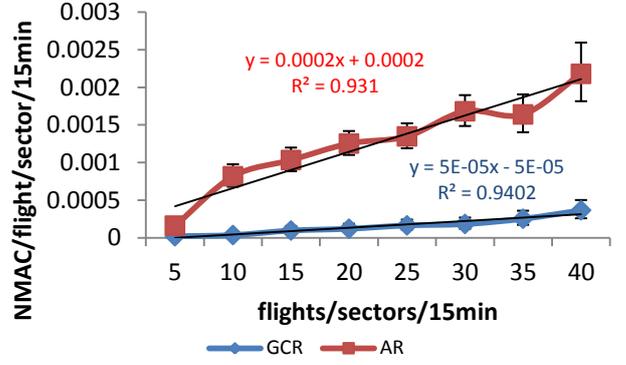
Fig 12.NMAC per flight per sector per fifteen minute, ZAU super high sectors
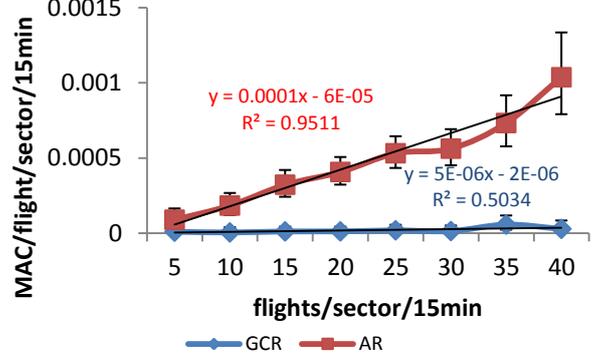
Fig 13. MAC per flight per sector per fifteen minute, ZAU super high sectors

The figures show confidence intervals from the fifty simulation runs. The confidence intervals show that some of the noise in these figures is due to limited simulation time. Because NMACs and MACs are rare events, we use confidence intervals based on the Poisson distribution instead of the normal distribution. The sum of a large number of independent rare events approximately follows a Poisson distribution [25]. The Poisson confidence interval is given by,

$$[H_{2X}^{-1}(\propto/2)/2, \ H_{2X+2}^{-1}(1-\propto/2)/2] \qquad (6)$$

where, $H_k(x)$ is the cumulative distribution function of a χ^2 distribution with k degrees of freedom, X is the total number of NMACs or MACs observed for given range of flight count, and 1-α is the desired confidence (e.g., α = .05 for a 95% confidence interval).

C. Rate of Mid Air Collision

In this section, we use analytical extensions to estimate the rate of MACs, as discussed in Section III. Fig 14 and Fig 15 show the comparison of NMAC and MAC rates before and after the application of the CPA algorithm to GCR. Applying the CPA algorithm increases the rate of NMAC and MAC by 2.5 and 5 times respectively.
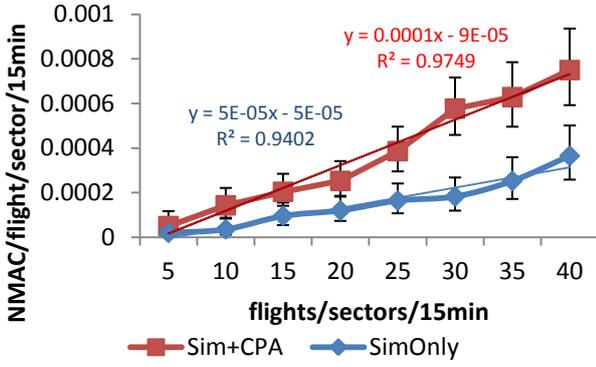
Fig 14 NMAC rate before and after CPA algorithm for GCR
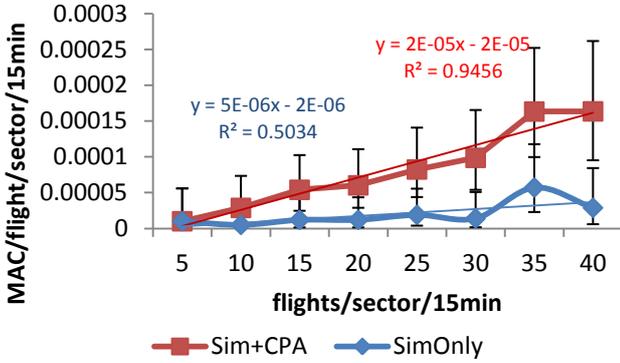
Fig 15 MAC rate before and after CPA algorithm for GCR

Fig **16** summarizes various methods for estimating MAC rates.
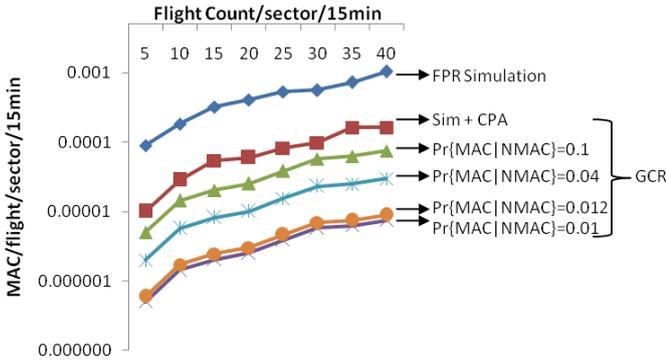
Fig 16. Rate of MAC, Comparison

The top line in the figure shows the MAC rate for *airway routes* based on a *pure simulation* approach (FACET output). The next line shows the MAC rate for *great-circle routes* obtained from simulation and the CPA algorithm. Thus, there are more MACs associated with airway routes than with great-circle routes, even when the CPA algorithm is applied to add missing MACs to the great-circle routes.

The next four lines show the MAC rates for great-circle routes using the analytical model in (3) and the conditional probabilities in Table 3. These lines come from a hybrid approach in which NMAC rates are obtained from simulation and these are extended to MAC rates via (3). The main

conclusion is that the hybrid analytical approach from (3) appears to underestimate the rate of MACs, probably because flights are assumed to be distributed uniformly in space, at least for equations (4) and (5). In the simulation, the structured nature of the routes and the corresponding intersections leads to a higher MAC probability.

V. CONCLUSION

This paper estimated rates of potential conflicts as a function of flight counts. We obtained estimates of conflict rates using FACET simulations of a 1.5X traffic scenario. Four different conflict volumes were considered (LOS, CLOS, NMAC, and MAC). A hybrid analytic approach was also used to extend the simulation results for NMAC and MAC rates. In all cases, conflict rates were estimated *under the assumption of no conflict resolution*. In other words, the results represented the rates at which aircraft were *on course* for a conflict (initiating events for a LOS, CLOS, NMAC, or MAC) but would not necessarily have resulted in a conflict.

The results of this analysis can be used to evaluate safety-capacity tradeoffs for future conflict detection and resolution (CD&R) automation concepts such as AAC and AFM. The probability of collision can be defined as the product of two terms, as shown in the analytical model (7). This analysis estimates the first term as function of sector capacity. In [26] and [27] the authors use dynamic event tree to estimate the second term based on the failure rates of the various components of AAC and AFM. By combining the two, an estimate of safety (in terms of probability of collision) as a function of sector capacity can be obtained.

$$\Pr\{Collision\}$$
$$= \Pr\{Aircraft\ on\ course\ for\ Conflict\} \qquad (7)$$
$$* \Pr\{Collision\ |\ Aircraft\ on\ course\ for\ conflict\}$$

One major observation from this work is that the estimated rates of NMACs and MACs are much higher for airway routes compared with great-circle routes. This indicates that if aircraft fly their user-preferred shortest distance, the resulting rate at which aircraft are on course for an NMAC or MAC decreases. Thus, automation has the potential to reduce not only the failure rate of the conflict detection and resolution itself, but also the rate of the initiating events in which aircraft get into a conflict in the first place. This work also highlighted the importance of simulating the underlying route structure, since MAC rates estimated using hybrid analytical approaches appeared to underestimate the rates obtained through pure simulation. Finally, this work confirmed the theoretical quadratic relationship between conflict rates and traffic counts.

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